Tennessee Technological University
Mathematics Department

MATH 6110-6120: Abstract Algebra I-II

I. COURSE DESCRIPTION FROM CATALOG:

An extensive treatment of groups, semigroups, integral domains, rings and ideals, fields and Galois fields. Lec. 3. Cr. 3.

II. PREREQUISITE(S):

MATH 6110: C or better in MATH 4010/5010 or consent of instructor.
MATH 6120: C or better in MATH 4020/5020 and C or better in MATH 6110, or consent of instructor.

III. COURSE OBJECTIVE(S):

To expand in-depth and breadth upon one-year undergraduate sequence in Abstract Algebra. New topics normally not covered in undergraduate sequences could include matrix algebras, free groups, generators and relations, bilinear forms, spectral theorem for normal operators, examples of algebras such as Clifford algebras, group algebras, Hecke algebras, or Lie algebras, linear groups, topics from group representations, algebraic geometry, quadratic fields, prime ideals, modules, the Structure Theorem for Abelian Groups, field extensions, Galois theory.

IV. STUDENT LEARNING OUTCOMES:

MATH 6110:

Upon successful completion of the course, a student will

- be familiar with basic properties of specific types of groups, including abelian groups, alternating groups, automorphism groups, cyclic groups, dihedral groups, matrix groups, the quaternion group, and symmetric groups;
- be able to determine various types of subgroups, including centers, centralizers, finitely generated subgroups, normal subgroups, normalizers, stabilizers, and Sylow *p*- subgroups;
- be comfortable applying major results—including, Cauchy's theorem, the class equation, the fundamental theorem of finitely generated abelian groups, the isomorphism theorems, Lagrange's theorem, the orbit-stabilizer theorem, and the Sylow theorems—to establish facts with respect to certain types of groups;
- have developed a working knowledge of free groups, group presentations, nilpotent groups, and solvable groups;
- be able to synthesize concepts and results in group theory to prove facts concerning groups

MATH 6120:

Upon successful completion of the course, a student will

- understand important concepts related to the theory of both commutative and noncommutative rings, including characteristic of a ring, ideals, factor rings, polynomial rings, quotient rings, and ring homomorphisms;
- be well versed in factorization-type properties of (integral) domains, particularly

those properties related to the concepts of Euclidean domain, principal ideal domain, and unique factorization domain;

- understand the basic algebraic structure of a module, and identify certain consequences of the finite generation of a module and the property of being a free module;
- know the connection between roots of polynomials and certain types of field extensions, including algebraic, algebraically closed, cyclotomic, finite, primitive, separable, and splitting field extensions;
- have developed a working knowledge of Galois theory, including the ability to apply the fundamental theorem of Galois theory to determine a Galois group;
- be able to synthesize concepts and results in ring theory to prove facts concerning rings

V. TOPICS TO BE COVERED:

MATH 6110:

1. Groups: definition, examples, subgroups, isomorphisms, homomorphisms, equivalence relations and partitions, cosets, restriction of homomorphisms to a subgroup, products of groups, quotient groups.
2. Vector spaces over fields: definition, bases, dimensions, direct sums.
3. Linear transformations: kernel, image, the dimension formula, matrix of a linear transformation, characteristic and minimal polynomials, eigenvalues/eigenvectors, diagonalization, matrix exponentiation.
4. Symmetry: symmetry of plane figures, the group of motions of the plane, finite groups of motions, discrete groups of motions, operations on cosets, the Counting Formula, permutation representations.
5. More group theory: automorphisms, the Sylow Theorems, the free group, generators and relations, the Todd-Coxeter Algorithm.

Optional selected topics, if time permits:

6. Bilinear Forms.
7. Linear Groups.
8. Group Representations.

MATH 6120:

9. Rings: definition, examples, polynomial rings, homomorphisms, ideals, quotient rings, integral domains, fraction fields, maximal ideals.
10. Factorization: factorization of integers and polynomials, unique factorization domains, principal ideal domains, Euclidean domains, Gauss's Lemma, Gaussian integers, algebraic integers, quadratic fields, ideal factorization.
11. Modules: definitions, examples, free modules, bases, generators and relations for modules, the Structure Theorem for Abelian Groups, free modules over polynomial rings.
12. Fields: examples, algebraic and transcendental elements, field extensions, ruler and compass constructions, adjunction of roots, finite fields, transcendental extensions.

13. Galois Theory: the Main Theorem, cubic equations, symmetric functions, primitive elements, proof of the Main Theorem, quartic equations, Kummer extensions, cyclotomic extensions.

VI. ADDITIONAL INFORMATION:

Lectures, homework, possibly lab assignments

VII. POSSIBLE TEXTS AND REFERENCES:

Abstract Algebra, Dummit and Foote, 3rd edition
Algebra by Michael Artin, Prentice Hall, Englewood Cliffs
A First Course in Abstract Algebra (2nd Edition) by Joseph J. Rotman, Prentice Hall, 2nd edition (February 28, 2000)
Advanced Modern Algebra by Joseph J. Rotman, Publisher: Prentice Hall, 1st edition (April 2002)

VIII. ANY TECHNOLOGY THAT MAY BE USED:

XI. STUDENT ACADEMIC MISCONDUCT POLICY

Maintaining high standards of academic integrity in every class at Tennessee Tech is critical to the reputation of Tennessee Tech, its students, alumni, and the employers of Tennessee Tech graduates. The Student Academic Misconduct Policy describes the definitions of academic misconduct and policies and procedures for addressing Academic Misconduct at Tennessee Tech. For details, view the Tennessee Tech's Policy 217 – Student Academic Misconduct at Policy Central.

X. DISABILITY ACCOMMODATION

Students with a disability requiring accommodations should contact the Accessible Education Center (AEC). An Accommodation Request (AR) should be completed as soon as possible, preferably by the end of the first week of the course. The AEC is located in the Roaden University Center, Room 112; phone 931-372-6119. For details, view the Tennessee Tech's Policy 340 – Services for Students with Disabilities at Policy Central.