CHAPTER 3

MODEL OF A THREE-PHASE INDUCTION MOTOR

3.1 Introduction

The induction machine is used in wide variety of applications as a means of converting electric power to mechanical power. Pump steel mill, hoist drives, household applications are few applications of induction machines. Induction motors are most commonly used as they offer better performance than other ac motors.

In this chapter, the development of the model of a three-phase induction motor is examined starting with how the induction motor operates. The derivation of the dynamic equations, describing the motor is explained. The transformation theory, which simplifies the analysis of the induction motor, is discussed. The steady state equations for the induction motor are obtained. The basic principles of the operation of a three phase inverter is explained, following which the operation of a three phase inverter feeding a induction machine is explained with some simulation results.

3.2. Basic Principle Of Operation Of Three-Phase Induction Machine

The operating principle of the induction motor can be briefly explained as, when balanced three phase voltages displaced in time from each other by angular intervals of 120° is applied to a stator having three phase windings displaced in space by 120° electrical, a rotating magnetic field is produced. This rotating magnetic field has a uniform strength and rotates at the supply frequency, the rotor that was assumed to be standstill until then, has electromagnetic forces induced in it. As the rotor windings are short circuited, currents start circulating in them, producing a reaction. As known from Lenz's law, the reaction is to counter the source of the rotor currents. These currents would become zero when the rotor starts rotating in the same direction as that of the rotating magnetic field, and with the same strength. Thus the rotor starts rotating trying to catch up with the rotating magnetic field. When the differential speed between these two become zero then the rotor currents will be zero, there will be no emf resulting in zero torque production. Depending on the shaft load the rotor will always settle at a speed ω_r, which is less than the supply frequency ω_e. This differential speed is called the slip speed ω_{so}. The relation between, ω_e and ω_{so} is given as [13]

$$\omega_{so} = \omega_e - \omega_r \tag{3.1}$$

If ω_m is the mechanical rotor speed then

$$\omega_r = \frac{P}{2}\omega_m . \tag{3.2}$$

3.3 Derivation Of Three-Phase Induction Machine Equations

The winding arrangement of a two-pole, three-phase wye-connected induction machine is shown in Figure 3.1. The stator windings of which are identical, sinusoidally distributed in space with a phase displacement of 120°, with N_s equivalent turns and resistance r_s.

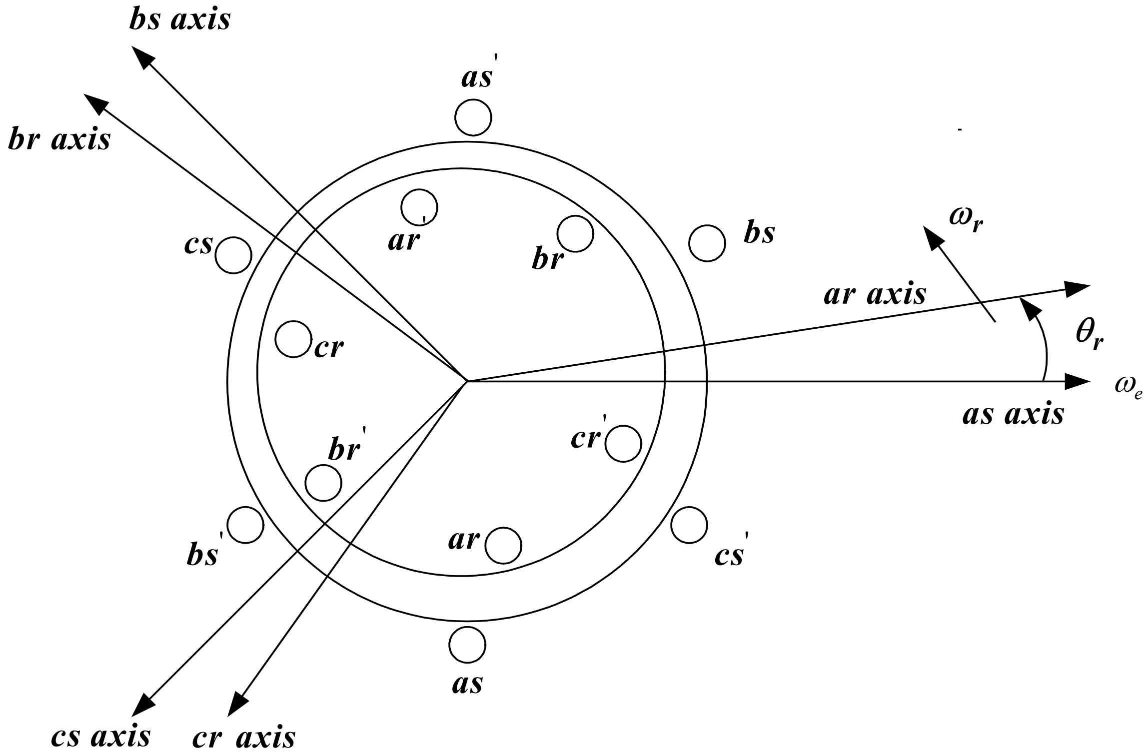

Figure 3.1. Two-pole three-phase symmetrical induction machine.

The rotor is assumed to symmetrical with three phase windings displaced in space by an angle of 120°, with N_r effective turns and a resistance of r_r. The voltage equations for the stator and the rotor are as given in Equations 3.3 to 3.8.

For the stator:

$$V_{as} = r_s I_{as} + p\lambda_{as} \tag{3.3}$$

$$V_{bs} = r_s I_{bs} + p\lambda_{bs} \tag{3.4}$$

$$V_{cs} = r_s I_{cs} + p\lambda_{cs} \tag{3.5}$$

where V_{as}, V_{bs} , and V_{cs} are the three phase balanced voltages which rotate at the supply frequency.For the rotor the flux linkages rotate at the speed of the rotor, which is ω_r:

$$V_{ar} = r_r I_{ar} + p\lambda_{ar} \tag{3.6}$$

$$V_{br} = r_r I_{br} + p\lambda_{br} \tag{3.7}$$

$$V_{cr} = r_r I_{cr} + p\lambda_{cr} \ . \tag{3.8}$$

The above equations can be written in short as

$$V_{abcs} = r_s I_{abcs} + p\lambda_{abcs} \tag{3.9}$$

$$V_{abcr} = r_r I_{abcr} + p\lambda_{abcr} \tag{3.10}$$

where

$$(V_{abcs})^T = [V_{as} \ V_{bs} \ V_{cs}] \tag{3.11}$$

$$(V_{abcr})^T = [V_{ar} \ V_{br} \ V_{cr}] \ . \tag{3.12}$$

In the above two equations 's' subscript denoted variables and parameters associated with the stator circuits and the subscript 'r' denotes variables and parameters associated with the rotor circuits. Both r_s and r_r are diagonal matrices each with equal nonzero elements. For a magnetically linear system, the flux linkages may be expressed as

$$\begin{bmatrix} \lambda_{abcs} \\ \lambda_{abcr} \end{bmatrix} = \begin{bmatrix} L_s & L_{sr} \\ (L_{sr})^T & L_r \end{bmatrix} \begin{bmatrix} i_{abcs} \\ i_{abcr} \end{bmatrix} \tag{3.13}$$

The winding inductances can be derived [16] and in particular

$$L_s = \begin{bmatrix} L_{ls} + L_m & -\frac{1}{2}L_m & -\frac{1}{2}L_m \\ -\frac{1}{2}L_m & L_{ls} + L_m & -\frac{1}{2}L_m \\ -\frac{1}{2}L_m & -\frac{1}{2}L_m & L_{ls} + L_m \end{bmatrix} \qquad (3.14)$$

$$L_r = \begin{bmatrix} L_{lr} + L_m & -\frac{1}{2}L_m & -\frac{1}{2}L_m \\ -\frac{1}{2}L_m & L_{lr} + L_m & -\frac{1}{2}L_m \\ -\frac{1}{2}L_m & -\frac{1}{2}L_m & L_{lr} + L_m \end{bmatrix} \qquad (3.15)$$

$$L_{sr} = L_{sr} \begin{bmatrix} \cos\theta_r & \cos(\theta_r + \frac{2\pi}{3}) & \cos(\theta_r - \frac{2\pi}{3}) \\ \cos(\theta_r - \frac{2\pi}{3}) & \cos\theta_r & \cos(\theta_r + \frac{2\pi}{3}) \\ \cos(\theta_r + \frac{2\pi}{3}) & \cos(\theta_r - \frac{2\pi}{3}) & \cos\theta_r \end{bmatrix} . \qquad (3.16)$$

In the above inductance equations, L_{ls} and L_m are the leakage and magnetizing inductances of the stator windings; L_{lr} and L_m are for the rotor windings. The inductance L_{sr} is the amplitude of the mutual inductances between stator and rotor windings.

From the above inductance equations, it can be observed that the machine inductances are functions of the rotor speed, whereupon the coefficients of the differential equations which describe the behavior of these machines are time varying except when the rotor is at standstill. A change of variables is often used to reduce the complexity of these differential equations, which gives rise to the reference frame theory [16]. For the induction machine under balanced operating conditions the synchronous reference frame of transformation is employed in which the reference frame rotates with the same frequency as that of the supply frequency ω_e. The transformation matrix used for the synchronous reference frame transformation is

$$K_s(\theta_s)=\frac{2}{3}\begin{bmatrix} \cos\theta_s & \cos(\theta_s-\frac{2\pi}{3}) & \cos(\theta_s+\frac{2\pi}{3}) \\ \sin\theta_s & \sin(\theta_s-\frac{2\pi}{3}) & \sin(\theta_s+\frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{3.17}$$

where $\theta_s=\int\omega_s\, t+\theta_{so}$, θ_{so} being the initial angle of the reference frame. The inverse transformation for the above transformation is given as

$$(K_s(\theta_s))^{-1}=\begin{bmatrix} \cos\theta_s & \sin\theta_s & 1 \\ \sin(\theta_s-\frac{2\pi}{3}) & \cos(\theta_s-\frac{2\pi}{3}) & 1 \\ \cos(\theta_s+\frac{2\pi}{3}) & \sin(\theta_s+\frac{2\pi}{3}) & 1 \end{bmatrix}. \tag{3.18}$$

The new variables after the transformation are related to the original variables as

$$f_{qdo}=(K_s(\theta_s))\ f_{abc} \tag{3.19}$$

where f can be voltage current flux or any thing. For the equations in 3.9 and 3.10 after transforming them in to the synchronous reference frame these equations become

$$V_{qs}=r_s i_{qs}+p\lambda_{qs}+\omega_e\lambda_{ds} \tag{3.20}$$

$$V_{ds}=r_s i_{ds}+p\lambda_{ds}-\omega_e\lambda_{qs} \tag{3.21}$$

$$V_{os}=r_s i_{os}+p\lambda_{os} \tag{3.22}$$

$$V_{qr}=r_r i_{qr}+p\lambda_{qr}+(\omega_e-\omega_r)\lambda_{dr} \tag{3.23}$$

$$V_{dr} = r_r i_{dr} + p\lambda_{dr} - (\omega_e - \omega_r)\lambda_{qr} \tag{3.24}$$

$$V_{or} = r_s i_{or} + p\lambda_{or} \tag{3.25}$$

where $V_{qs}, I_{qs}, \lambda_{qs}$ are the q-axis components, $V_{ds}, I_{ds}, \lambda_{ds}$ are the d-axis components, and $V_{os}, I_{os}, \lambda_{os}$ belong to the 0- axis and usually represent the unbalances in the system. In case of balanced voltages the zero-axis currents, voltages and flux are zero under normal operating conditions. The flux linkage equations expressed in abc variables given in Equation 3.13 yields the flux linkage equations for a magnetically linear system

$$\begin{bmatrix} \lambda_{qd0s} \\ \lambda_{qd0r} \end{bmatrix} = \begin{bmatrix} K_s L_s (K_s^{-1}) & K_s L_{sr} (K_s^{-1}) \\ K_s L_{sr} (K_s^{-1}) & K_s L_r (K_s^{-1}) \end{bmatrix} \begin{bmatrix} i_{qd0s} \\ i_{qd0r} \end{bmatrix} \tag{3.26}$$

where $L_s, L_r, and\ L_{sr}$ are as defined in Equations 3.14 to 3.16. Evaluating each term in the matrix of Equation 3.26 we get

$$K_s L_s (K_s^{-1}) = \begin{bmatrix} L_{ls} + L_m & 0 & 0 \\ 0 & L_{ls} + L_m & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} \tag{3.27}$$

$$K_s L_r (K_s^{-1}) = \begin{bmatrix} L_{lr} + L_m & 0 & 0 \\ 0 & L_{lr} + L_m & 0 \\ 0 & 0 & L_{lr} \end{bmatrix} \tag{3.28}$$

$$K_s L_{sr} (K_s^{-1}) = \begin{bmatrix} L_m & 0 & 0 \\ 0 & L_m & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.27}$$

From above the flux linkages the expressions are modified as

$$\lambda_{qs} = L_s i_{qs} + L_m i_{qr} \tag{3.28}$$

$$\lambda_{ds} = L_s i_{ds} + L_m i_{dr} \tag{3.29}$$

$$\lambda_{0s} = L_{ls} i_{0s} \tag{3.28}$$

$$\lambda_{qr} = L_r i_{qr} + L_m i_{qs} \tag{3.30}$$

$$\lambda_{dr} = L_r i_{dr} + L_m i_{ds} \tag{3.31}$$

$$\lambda_{0r} = L_{lr} i_{0r} \tag{3.32}$$

The expression for the electromagnetic torque in terms of the reference frame variables can be expressed as

$$T_e = (\frac{P}{2})[(K_s)^{-1} i_{qdos}]T \frac{\partial}{\partial \theta_r}[L_{sr}](K_s)^{-1} i_{qdor} \tag{3.33}$$

The torque expression in Equation 3.33 can be expressed in terms of currents as

$$T_e = (\frac{3}{2})(\frac{P}{2}) L_m (i_{qs} i_{dr} - i_{ds} i_{qr}) \tag{3.34}$$

where T_e is positive for motor action. Alternative expressions for the torque can be expressed in terms of flux linkages are given in Equations 3.35 and 3.36.

$$T_e = (\frac{3}{2})(\frac{P}{2})(\lambda_{qr} i_{dr} - \lambda_{dr} i_{qr}) \tag{3.35}$$

$$T_e = (\frac{3}{2})(\frac{P}{2})(\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}) \tag{3.36}$$

The expression for the rotor speed is expressed in terms of torque as

$$p\omega_r = \frac{P}{2J}(T_e - T_L) \tag{3.37}$$

Where T_e, T_L are the electromagnetic and the load torque, respectively, depending on the load torque the motor settles at a speed, which is always less than the angular frequency with which it gets excited. The load torque should be positive for motoring operation of the induction machine.

3.3.1 Equivalent Circuit Of The Induction Machine

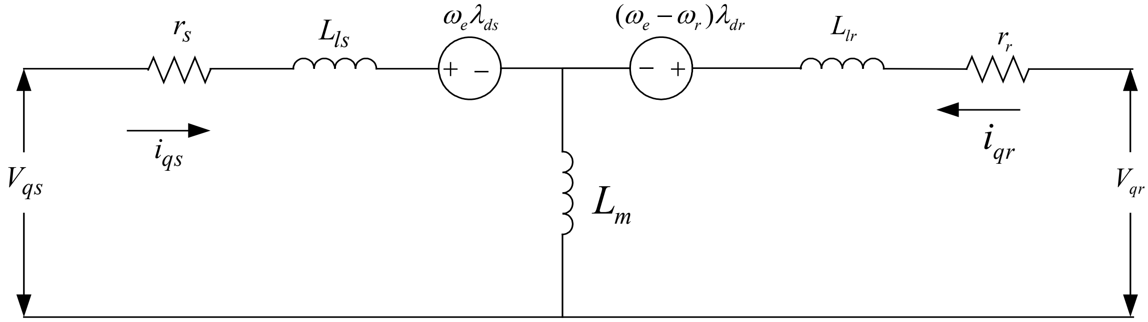

Figure 3.2 Arbitrary reference frame equivalent circuit for a 3-phase, symmetrical induction machine in the q-variables.

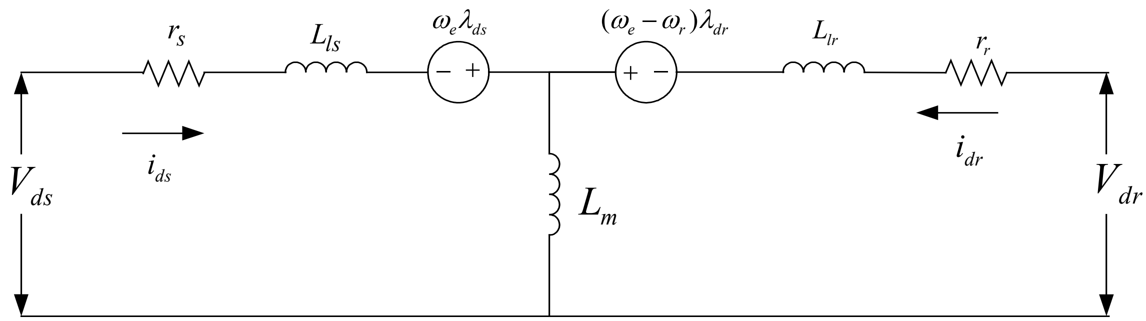

Figure 3.3 Arbitrary reference frame equivalent circuit for a 3-phase, symmetrical induction machine in the d-variables.

The Induction machine equivalent circuit is useful in the analysis of the machine when the motor settles at the rated speed. The equivalent circuit of the machine in the reference frame variables is formed using the equations from 3.20 to 3.25. The equivalent circuit for the stator is as shown in Figure 3.2 and that of the rotor is as shown in Figure 3.3.

3.4 Free Acceleration Characteristics

The variables of the induction machine during free acceleration or on no-load are observed which gives a deeper insight as to how the machine operates. The nonlinear differential equations as given in equations from 3.20 to 3.25 are used. The machine is simulated with either flux or currents as a state variable. Choosing flux as the state variable the equations are less complicated, thus flux is chosen to be the state variable and later from the flux equations the currents are obtained. The equations for the computer simulation in terms of flux linkages that is $\lambda_{qs}, \lambda_{ds}, \lambda_{qr}, \lambda_{dr}$ are given from 3.38 to 3.43.

$$p\lambda_{qs} = V_{qs} - \frac{r_s L_r}{K}\lambda_{qs} - \omega\lambda_{ds} + \frac{r_s L_m}{K}\lambda_{qr} \tag{3.38}$$

$$p\lambda_{ds} = V_{ds} - \frac{r_s L_r}{K}\lambda_{ds} - \omega\lambda_{qs} + \frac{r_r L_m}{K}\lambda_{dr} \tag{3.39}$$

$$p\lambda_{qr} = V_{qr} - \frac{r_r L_s}{K}\lambda_{qr} + \frac{r_r L_m}{K}\lambda_{qs} - (\omega - \omega_r)\lambda_{dr} \tag{3.40}$$

$$p\lambda_{dr} = V_{dr} - \frac{r_r L_s}{K}\lambda_{dr} + \frac{r_r L_m}{K}\lambda_{ds} + (\omega - \omega_r)\lambda_{qr} \tag{3.41}$$

$$Te = \frac{3P}{4}\frac{L_m}{L_r}(\lambda_{dr} i_{qs} - \lambda_{qr} i_{ds}) \tag{3.42}$$

$$p\omega_r = \frac{p}{2J}(T_e - T_L) \tag{3.43}$$

$$K = L_s L_r - L_m{}^2 \tag{3.44}$$

where ω reference frame speed, ω can be any reference frame, it can be synchronous ($\omega = \omega_e$), rotor ($\omega = \omega_r$) or the stator reference ($\omega = 0$) frame depending on which reference frame will make the analysis simple. Since the Induction machine is symmetric in both the stator and the rotor in the sense the stator and the rotor have the same kind of windings choosing any reference frame will not make any difference to simplify the analysis.

After solving for the flux expressions currents be obtained from them as

$$i_{qs} = \frac{1}{K}(L_r \lambda_{qs} - L_m \lambda_{qr}) \tag{3.45}$$

$$i_{ds} = \frac{1}{K}(L_r \lambda_{ds} - L_m \lambda_{dr}) \tag{3.46}$$

$$i_{qr} = \frac{1}{K}(L_s \lambda_{qr} - L_m \lambda_{qs}) \tag{3.47}$$

$$i_{dr} = \frac{1}{K}(L_s \lambda_{dr} - L_m \lambda_{ds}) \tag{3.48}$$

The induction machine whose parameters are given in Appendix A is simulated using Matlab/Simulink. The free acceleration characteristics are plotted in Figures 3.4 and 3.5, which show the load torque, rotor speed, the stator phase ‘a’ and ‘b’ currents, the rotor ‘phase ‘a’ current. In Figure 3.6 and 3.7 the a load of 4N-m was put on the machine for about 1 sec and then the plots of speed, torque and currents was plotted against time. It can be seen that when the load is added the speed drops down and when it is removed the speed builds up again to its rated value of 377 rad/sec as it is operating at 60 Hz supply frequency. The load was added on the machine from 0.5 to 1.5 sec and the currents have been magnified in this region.

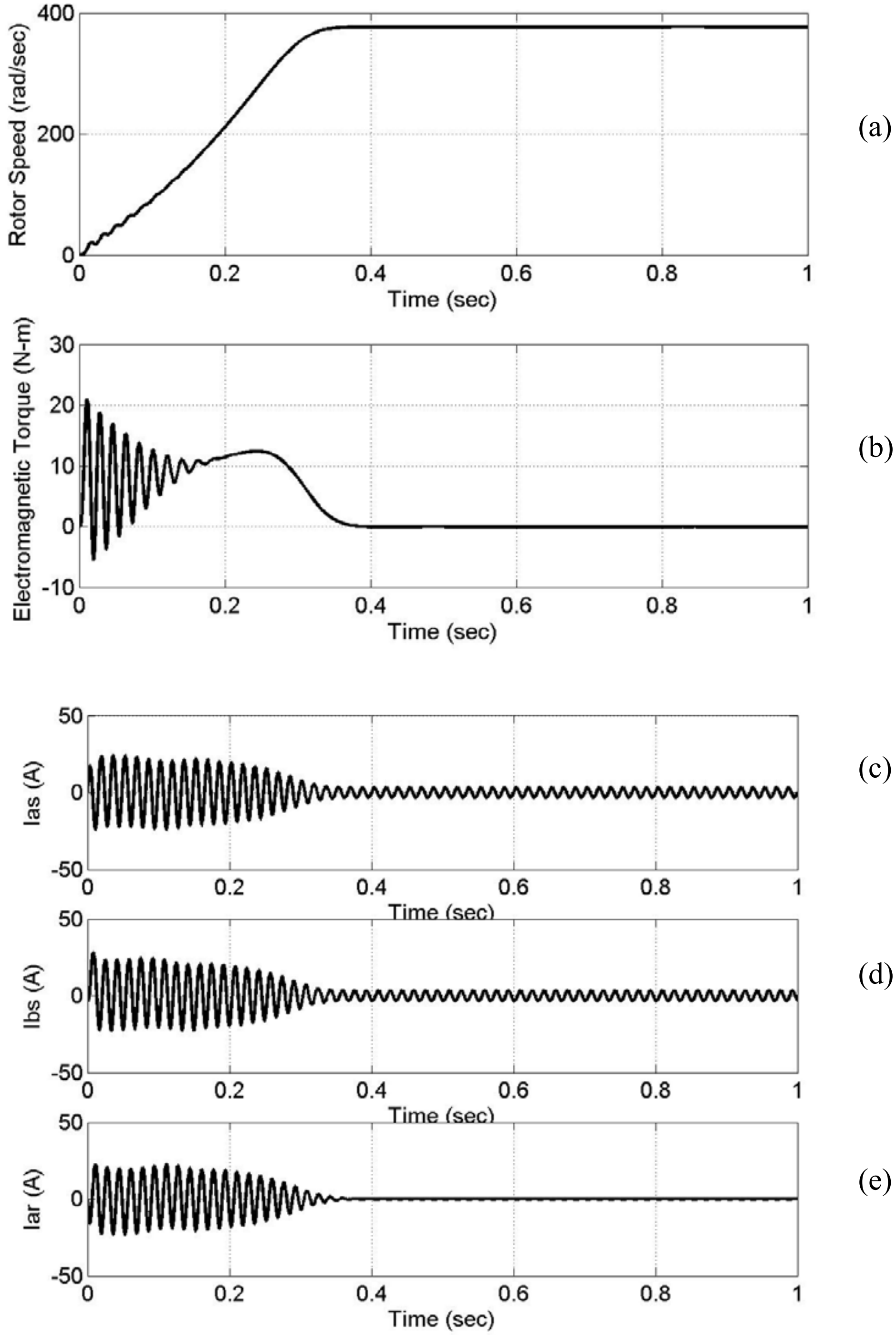

Figure 3.4. Free-acceleration characteristics, (a) rotor speed, (b) torque, (c) phase ‘a’ current (d) phase ‘b’ current (e) rotor phase ‘a’ current

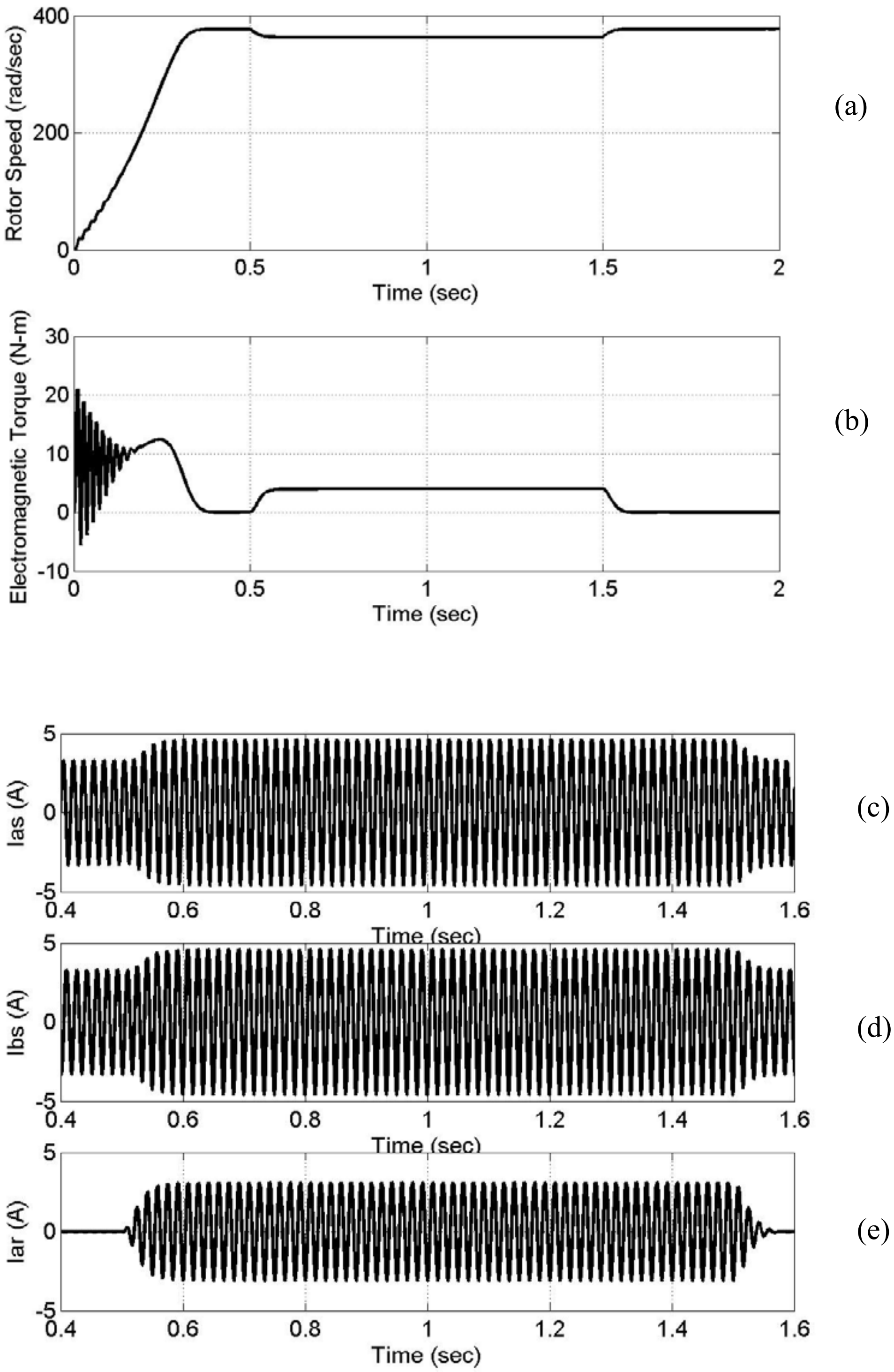

Figure 3.5. On-load characteristics, (a) rotor speed, (b) torque, (c) phase ‘a’ current (d) phase ‘b’ current (e) rotor phase ‘a’ current