CHAPTER 2

DERIVATION OF STATE EQUATIONS AND PARAMETER DETERMINATION OF AN IPM MACHINE

2.1 Derivation of Machine Equations

A model of a 3 phase PM machine is shown in Figure 2.1. Both the abc and the dq axes are shown in the figure. The magnetizing current, due to the presence of the magnet, is represented in Figure 2.1 by the current source i_f . The fictitious current source (along with the associated field winding L_{md}) will be used in the derivation to follow to obtain an expression for the flux which is equivalent to the flux created by the presence of the magnet

It is assumed in the following derivation that the rotor speed of the machine is constant . Thus, no current flows in the damper windings contained within the rotor and their presence may be ignored. It is also assumed that the machine is balanced.

The stator voltage equations of the IPM are [31]

$$\begin{aligned} v_a &= \frac{d}{dt}\lambda_a + i_a r_1 \\ v_b &= \frac{d}{dt}\lambda_b + i_b r_1 \\ v_c &= \frac{d}{dt}\lambda_c + i_c r_1 \ , \end{aligned} \tag{2.1}$$

where v_a , v_b , v_c are the a,b,c stator terminal voltages , λ_a , λ_b ,λ_c are the flux linkages in the abc plane, i_a , i_b ,i_c are the abc stator currents, and r_1 is the stator resistance of each phase

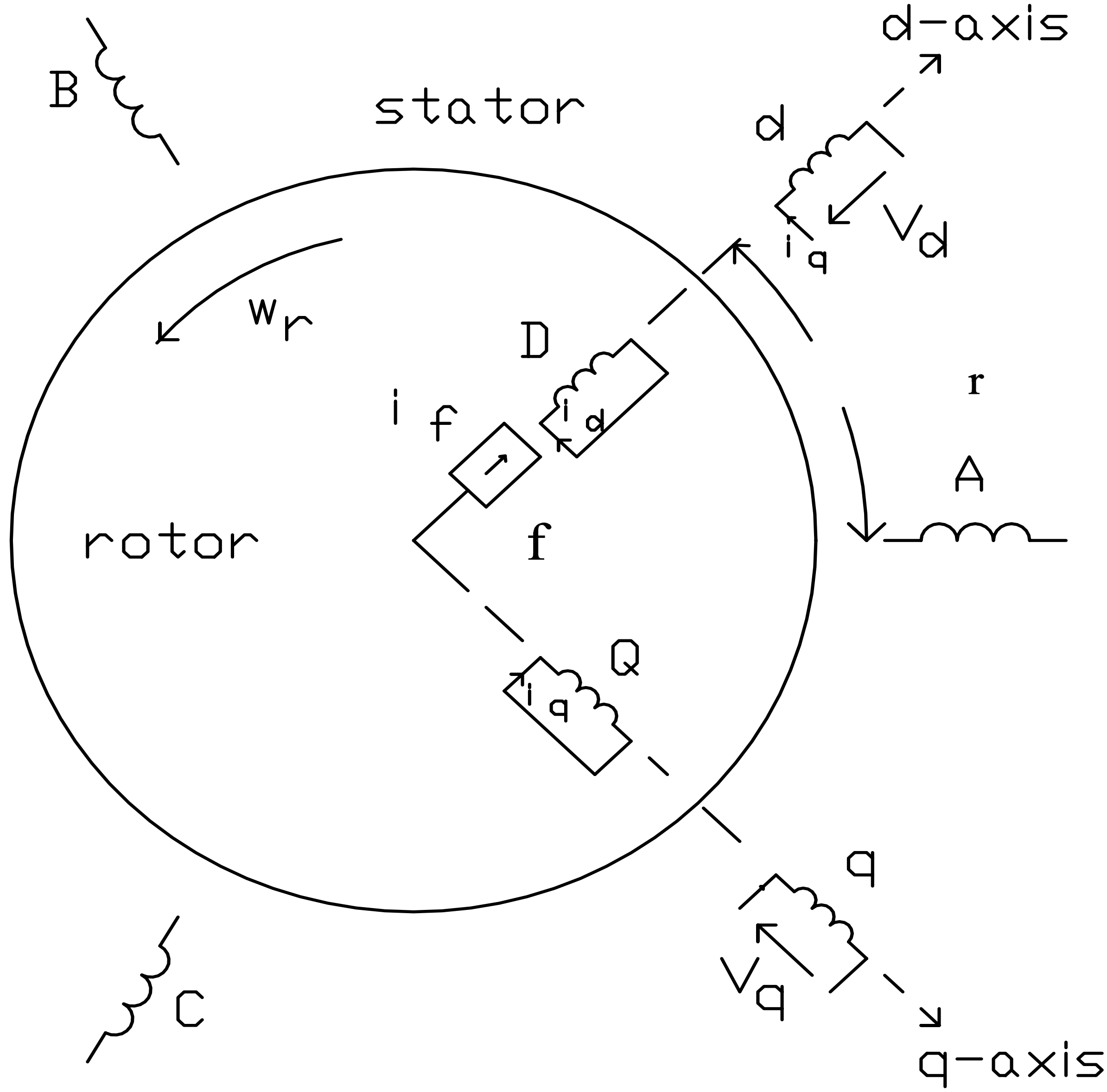

Figure 2.1. Model of a brushless PM synchronous machine

of the stator winding. Thus, the assumption has been made that the resistances for each phase of the stator are the same.

The flux linkages expressed in terms of the stator and terms of the current are

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L_{afd} & 0 & 0 \\ L_{bfd} & 0 & 0 \\ L_{cfd} & 0 & 0 \end{bmatrix} \begin{bmatrix} i_f \\ 0 \\ 0 \end{bmatrix} . \qquad (2.2)$$

The inductances given in Equation (2.2) will be kept in their symbolic form, transformed into their equivalent dq axis counterparts, and then be defined in terms of the physical construction of machine.

Due to the salient nature of the rotor, the inductances of Equation (2.2) are functions of the position of the rotor and (assuming that the rotor is spinning) are functions of time. This means that the inductance parameters are constantly changing - making the analysis of the machine very difficult in its present form. A transformation commonly referred to as Park's transformation allows the equations describing the machine to be transformed into a reference frame where the inductances are not functions of time. The reference frame chosen to accomplish this goal is dependent upon the type of machine being looked at. For a synchronous machine, this reference frame is that of the rotor. In other words, the stator voltages, currents, and inductances will be projected onto the rotor side of the machine and, thus, be rotating at the same speed with which the rotor is spinning. With this accomplished, the inductances no longer vary with the position of the rotor.

The transformation of a 3 phase balanced voltage, current, or flux linkage from the

$$h_{qdo} = T(\theta)\, h_{abcs} \ , \tag{2.3}$$

abc to the dq reference plane may be expressed as [32]

$$T(\theta) = \frac{2}{3}\begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin(\theta) & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} ,$$

where

and θ denotes the reference frame chosen. For the rotor reference frame, $\theta=\theta_r$. Expressing the q d and o terms individually and substituting $\theta=\theta_r$ gives

$$\begin{aligned} h_q &= \frac{2}{3}[\, h_a \cos(\theta_r) + h_b \cos(\theta_r - \frac{2\pi}{3}) + h_c \cos(\theta_r + \frac{2\pi}{3})] \\ h_d &= \frac{2}{3}[\, h_a \sin(\theta_r) + h_b \sin(\theta_r - \frac{2\pi}{3}) + h_c \sin(\theta_r + \frac{2\pi}{3})] \\ h_o &= \frac{1}{3}[\, h_a + h_b + h_c] \ , \end{aligned} \tag{2.4}$$

where, for balanced conditions, $h_0 = 0$ and the o term will, from this point forward, be ignored. So, using Equation (2.4), the flux linkages may be expressed in the dq plane as

$$\lambda_q = \frac{2}{3}[\lambda_a \cos(\theta_r) + \lambda_b \cos(\theta_r - \frac{2\pi}{3}) + \lambda_c \cos(\theta_r + \frac{2\pi}{3})]$$
$$\lambda_d = \frac{2}{3}[\lambda_a \sin(\theta_r) + \lambda_b \sin(\theta_r - \frac{2\pi}{3}) + \lambda_c \sin(\theta_r + \frac{2\pi}{3})] \quad . \qquad (2.5)$$

$$\frac{d\lambda_q}{dt} = -\frac{2}{3}[\lambda_a \sin(\theta_r)\frac{d\theta_r}{dt} + \lambda_b \sin(\theta_r - \frac{2\pi}{3})\frac{d\theta_r}{dt} + \lambda_c \sin(\theta_r + \frac{2\pi}{3})\frac{d\theta_r}{dt}]$$
$$+ \frac{2}{3}[\frac{d\lambda_a}{dt}\cos(\theta_r) + \frac{d\lambda_b}{dt}\cos(\theta_r - \frac{2\pi}{3}) + \frac{d\lambda_c}{dt}\cos(\theta_r + \frac{2\pi}{3})] \quad . \qquad (2.6)$$

Taking the derivative of λ_q in Equation (2.5) with respect to time gives

Comparing the first bracketed expression of Equation (2.6) with the expression for λ_d in Equation (2.5) , gives

$$-\lambda_d \frac{d\theta_r}{dt} = [\lambda_a \sin(\theta_r)\frac{d\theta_r}{dt} + \lambda_b \sin(\theta_r - \frac{2\pi}{3})\frac{d\theta_r}{dt} + \lambda_c \sin(\theta_r + \frac{2\pi}{3})\frac{d\theta_r}{dt}] \quad . \qquad (2.7)$$

Rearranging the voltage equation of (2.1) in terms of the rate of change of flux linkage gives

$$\frac{d\lambda_a}{dt} = v_a - i_a r_1$$
$$\frac{d\lambda_b}{dt} = v_b - i_b r_1 \qquad (2.8)$$
$$\frac{d\lambda_c}{dt} = v_c - i_c r_1 \quad .$$

Substituting Equation (2.8) into the second bracketed expression of (2.6) gives

$$\frac{d\lambda_a}{dt}\cos(\theta_r)+\frac{d\lambda_b}{dt}\cos(\theta_r-\frac{2\pi}{3})+\frac{d\lambda_c}{dt}\cos(\theta_r+\frac{2\pi}{3})=$$
$$[v_a\cos(\theta_r)+v_b\cos(\theta_r-\frac{2\pi}{3})+v_c\cos(\theta_r+\frac{2\pi}{3})]\ -$$
$$[i_a r_1\cos(\theta_r)+i_b r_1\cos(\theta_r-\frac{2\pi}{3})+i_c r_1\cos(\theta_r+\frac{2\pi}{3})]$$
$$=v_{qs}-i_{qs}r_1 \qquad (2.9)$$

Substituting the results of Equations (2.7) and (2.9) into (2.6) gives

$$\frac{d\lambda_q}{dt}=-\lambda_d\frac{d\theta_r}{dt}+v_{qs}-i_{qs}r_1\ . \qquad (2.10)$$

Letting

$$p=\frac{d}{dt}\ ,\ and\ \omega_r=\frac{d\theta_r}{dt}\ ,$$

then

$$p\lambda_q=-\lambda_d\omega_r+v_{qs}-i_{qs}r_1\ . \qquad (2.11)$$

Therefore, the q-axis voltage can be written as

$$v_{qs}=p\lambda_q+i_{qs}r_1+\lambda_d\omega_r\ . \qquad (2.12)$$

Similarly, taking the derivative of the d-axis flux linkage of Equation (2.5) gives

$$\frac{d\,\lambda_d}{dt} = \frac{2}{3}[\,\lambda_a \cos(\theta_r)\frac{d\,\theta_r}{dt} + \lambda_b \cos(\theta_r - \frac{2\,\pi}{3})\frac{d\,\theta_r}{dt} + \lambda_b \cos(\theta_r + \frac{2\,\pi}{3})\frac{d\,\theta_r}{dt}\,]$$
$$+ \frac{2}{3}[\,\frac{d\,\lambda_a}{dt}\sin(\theta_r) + \frac{d\,\lambda_b}{dt}\sin(\theta_r - \frac{2\,\pi}{3}) + \frac{d\,\lambda_c}{dt}\sin(\theta_r + \frac{2\,\pi}{3})] \;. \qquad (2.13)$$

$$\lambda_q \frac{d\,\theta_r}{dt} = [\,\lambda_a \cos(\theta_r)\frac{d\,\theta_r}{dt} + \lambda_b \cos(\theta_r - \frac{2\,\pi}{3})\frac{d\,\theta_r}{dt} + \lambda_c \cos(\theta_r + \frac{2\,\pi}{3})\frac{d\,\theta_r}{dt}\,] \;. \qquad (2.14)$$

Comparing the first bracketed expression in Equation (2.13) with the equation for q axis flux linkage in Equation (2.5) gives the result

Substituting the results of Equation (2.8) into the second bracketed expression of Equation (2.13) gives

$$\frac{d\,\lambda_a}{dt}\sin(\theta_r) + \frac{d\,\lambda_b}{dt}\sin(\theta_r - \frac{2\,\pi}{3}) + \frac{d\,\lambda_c}{dt}\sin(\theta_r + \frac{2\,\pi}{3}) =$$
$$[\,v_a \sin(\theta_r) + v_b \sin(\theta_r - \frac{2\,\pi}{3}) + v_c \sin(\theta_r + \frac{2\,\pi}{3})] -$$
$$[\,i_a\, r_1 \sin(\theta_r) + i_b\, r_1 \sin(\theta_r - \frac{2\,\pi}{3}) + i_c\, r_1 \sin(\theta_r + \frac{2\,\pi}{3})] \qquad (2.15)$$

$$= v_{ds} - i_{ds}\, r_1 \,.$$

Substituting the results of Equations (2.14) and (2.15) into Equation (2.13) gives

$$\frac{d\,\lambda_d}{dt} = \lambda_q \frac{d\,\theta_r}{dt} + v_{ds} - i_{ds}\, r_1 \,. \qquad (2.16)$$

After rearranging Equation (2.16) and substituting Equation (2.10) into it, the final result for the d axis voltage is

$$v_{ds} = p\lambda_d + i_{ds} r_1 - \lambda_q \omega_r . \tag{2.17}$$

In order to obtain the individual self and mutual inductance terms (given in Equation (2.2)), the parameters may be defined in terms of the physical construction of the machine , i.e. number of windings, physical dimensions, etc. , and then the equations obtained may be transformed into the dq reference frame. Alternatively, as was done in this thesis, the flux linkage equations may be transformed directly into the dq rotor reference frame (θ equals θ_r) and the q and d axis inductances may be defined directly in that frame of reference. Thus,

$$\begin{bmatrix} \lambda_q \\ \lambda_d \\ \lambda_o \end{bmatrix} = T(\theta) \begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix} = T(\theta) \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix} (T(\theta))^{-1} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_o \end{bmatrix} + T(\theta) \begin{bmatrix} L_{afd} & L_{adr} & L_{aqr} \\ L_{bfd} & L_{bdr} & L_{bqr} \\ L_{cfd} & L_{cdr} & L_{cqr} \end{bmatrix} \begin{bmatrix} i_f \\ i_{dr} \\ i_{qr} \end{bmatrix}, \tag{2.18}$$

where

$$T(\theta) \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix} (T(\theta))^{-1} = \begin{bmatrix} L_{ls} + L_{mq} & 0 & 0 \\ 0 & L_{ls} + L_{md} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix},$$

and

$$T(\theta)\begin{bmatrix} L_{afd} & 0 & 0 \\ L_{bfd} & 0 & 0 \\ L_{cfd} & 0 & 0 \end{bmatrix} = \begin{bmatrix} L_{md} & 0 & 0 \\ 0 & L_{mdr} & 0 \\ 0 & 0 & L_{mqr} \end{bmatrix}.$$

It is worthwhile to reiterate that the field current i_f is fictitious and has been used to represent the flux linkage from the magnet source. From this point forth the term λ_e will be used instead of the product term $i_f * L_{md}$. Since the rotor speed is assumed constant, i_{dr} and i_{qr} are equal to zero.

The flux linkage equations in the dq axis can be expressed individually as

$$\begin{aligned} \lambda_d &= L_d\, i_{ds} + \lambda_e \\ \lambda_q &= L_q\, i_{qs} \ , \end{aligned} \tag{2.19}$$

where

$$\begin{aligned} L_d &= L_{ls} + L_{md} \\ L_q &= L_{ls} + L_{mq} \ , \end{aligned}$$

and L_{md} and L_{mq} are the d and q axis mutual inductances and L_{ls} is the leakage in the stator.

The d and q axis mutual inductances are given as [12]

$$L_{mq} = C_q L_m$$
$$L_{md} = C_d L_m \quad , \tag{2.20}$$

where L_m is the inductance of a machine with a uniform air gap and no magnets. This inductance is determined from the flux linking with $N_1 C_w$ effective turns, and is given as

$$L_m = 1.273\, \mu_o\, m_1 \left(\frac{N_1 C_w}{P}\right)^2 \frac{D_i L}{g} 10^{-8} H \quad , \tag{2.21}$$

where all the terms given in Equation (2.21) are in meter, kilogram, second (MKS) units

μ_o = permeability of free space

m_1 = number of phases of the machine

N_i = number of series turns per phase

C_w = a winding factor which is a product of the distribution and pitch factors

D_i = stator inner diameter

P = number of poles

L = core length

g = effective air gap length.

C_q and C_d are factors which account for the presence of the magnets and are, for an interior permanent magnet, given as

$$C_q = \rho - \sin\frac{\rho\pi}{\pi}$$

$$C_d = \rho + \sin\frac{\rho\pi}{\pi} - \frac{(\frac{8}{\rho}\pi^2)\sin^2\rho\frac{\pi}{2}}{1+\frac{R_g}{R_m}} \quad , \qquad (2.22)$$

where

ρ = the pole arc

R_g = reluctance of the air gap

R_m = reluctance of the magnet .

The open circuit magnet flux λ_e for an IPM machine is given as [12]

$$\lambda_e = \frac{4.44}{2\pi} N_i C_w \frac{(\pi D_i L)}{P} B_f^! * 10^{-8} \quad , \qquad (2.23)$$

where $B^!_f$ is the amplitude of the fundamental flux density created by an individual magnet.

In summary, the voltage and flux equations needed to analyze a permanent magnet under the stated assumptions are

$$v_{qs} = p\,\lambda_{qs} + i_{qs} r_s + \lambda_{ds}\,\omega$$
$$v_{ds} = p\,\lambda_{ds} + i_{ds} r_s - \lambda_{qs}\,\omega \quad , \qquad (2.24)$$

where

$$\lambda_{ds} = L_{ds} i_{ds} + \lambda_e$$
$$\lambda_{qs} = L_{qs} i_{qs} \quad ,$$

and the subscript s on the q and d axis stator terms has been added, r_s has replaced r_1 as the symbol used to represent the stator resistance, and the subscript r has been dropped from ω. Under steady-state, the derivatives of the state variables of Equation (2.24) are zero so, at steady state the voltage equations may be written as

$$V_{qs} = I_{qs} r_s + \lambda_{ds} \omega$$
$$V_{ds} = I_{ds} r_s - \lambda_{qs} \omega \; . \tag{2.25}$$

A d-q axis schematic diagram representing the equations given in (2.25) is shown in Figure 2.2

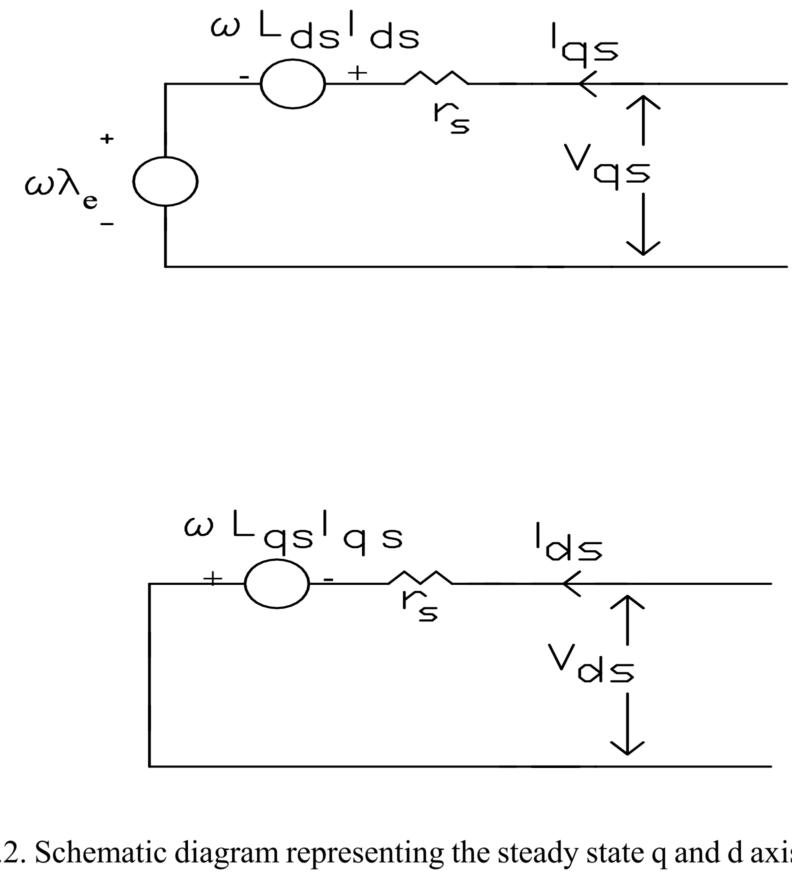

Figure 2.2. Schematic diagram representing the steady state q and d axis voltage equations of a PM machine

2.2 Determination of Parameters of the IPM

The single most important determination to be made in the analysis of an electrical machine is to find what the parameters of the machine are. Due to the effects of saturation at heavy load and demagnetization of the magnet at light loads, the parameters of the machine change significantly as the load presented to it changes. Thus, the parameters (namely the mutual inductances in the d and q axes and the flux linkage created by the magnet) must be functions of the operating conditions.

Generally, when effects due to saturation are included in an analysis, the parameters are made functions of either the total mutual flux λ_{mm} or the total stator current Is. The decision was made to make the parameters functions of stator peak current because, when the parameters were plotted as a function of flux, there were regions in which two possible values of a particular parameter could be obtained for a single value of flux (which could be problematic when solving equations).

The parameters are found by using the equations given in Equation (2.25) and repeated below for the sake of continuity.

$$V_{qs} = r_s I_{qs} + \omega L_{ds} I_{ds} + \omega \lambda_e$$

$$V_{ds} = r_s I_{ds} - \omega L_{qs} I_{qs} \quad . \tag{2.26}$$

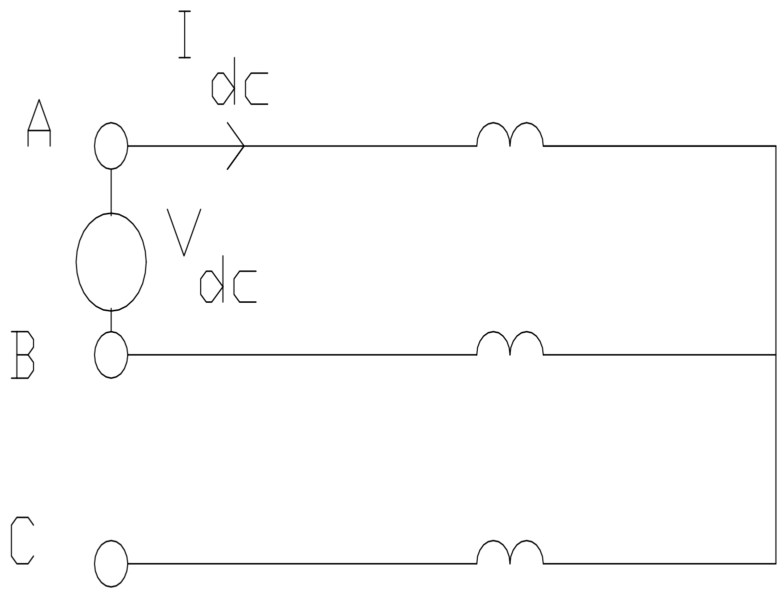

Figure 2.3. Schematic diagram of dc test used to determine stator resistance

The stator resistive value r_s was found by applying a dc voltage across two terminals of the stator and measuring both the voltage and the current which flowed through the terminals (see Figure 2.3). The stator resistance for a single phase is given as

$$r_s = \frac{V_{dc}}{2\, I_{dc}} \quad . \tag{2.27}$$

The voltages and currents in Equation (2.26) were found by varying a three phase balanced resistive load from a high value to a low value and recording the terminal voltage and the current output from the generator. The measurements made were the line to line voltages and the phase currents. In order to convert the voltages and currents into their dq components, the torque angle δ was needed. The power factor of the machine is also needed in parameter determination, but, since the generator was feeding a resistive load, the current out of the generator was in phase with the voltage at the terminals, so the power factor was unity. The torque angle was found by measuring the difference in angle of the voltages of a search coil located across phase "a" of the stator and the terminal voltage appearing at the stator terminal of phase "a." This method was not an ideal way to measure the torque angle because the oscilloscope used to measure the angle between the two voltages gave a varying readout even though the load and speed of the generator were constant. An average of the numbers was taken and used as the torque angle. It would have been much easier (and probably more accurate) to have a commercially available torque angle measuring device; however, no such device was available. Nevertheless, the strong corroboration between measured and predicted results suggests that the method used was an acceptable means of obtaining the torque angle.

Once the stator voltages, currents, and torque angle are known, the dq voltages and

$$\begin{aligned} V_{qs} &= V_s \operatorname{Cos}(\delta) \\ V_{ds} &= -V_s \operatorname{Sin}(\delta) \\ I_{qs} &= I_s \operatorname{Cos}(\gamma) \\ I_{ds} &= -I_s \operatorname{Sin}(\gamma) \;, \end{aligned} \tag{2.28}$$

currents can be found by the following relations:

where V_s is the peak line to neutral voltage, I_s is the peak stator current, and γ is the sum of the torque angle δ and the power factor angle θ. Since the power factor is unity (since the generator is feeding a purely resistive load), then γ is equal to δ.

With the dq voltages and currents and the stator resistance known, the inductance in the q axis can easily be found and is given as

$$L_{qs} = -\frac{V_{ds} - r_s I_{ds}}{\omega I_{qs}} \quad . \tag{2.29}$$

The inductance in the d axis and the magnet flux linkage are not as easy to find as the q axis inductance. The two terms are contained in the same equation and are, in a sense, coupled together.

One method of finding the magnet flux involves running a no load test on the PM machine for a range of frequencies and measuring the terminal voltage of the machine and the voltage across the terminals of the search coil. An empirical relationship between the search coil voltage and the magnet voltage (and thus the magnet flux linkage) can be developed since, at a no load condition, the terminal voltage of the machine is equal to the magnet voltage. Figure 2.4 shows a plot of the rms voltage of the magnet vs. the air gap voltage for both the series connection (high voltage) and the parallel connection (low voltage) of the stator winding of the PM machine.

The low voltage connection was not used in any of the experiments reported in this thesis (except for the one just described). The main reason for this is that, since the machine is being operated in generator mode, one would normally like a high terminal voltage and the low voltage connection is, as one would expect, one half of the terminal voltage of the high voltage connection for any particular operating frequency.

This brings up one other point of interest, which is that one would ideally like to have a generator having a magnet voltage greater than 120 volts rms when operating at 60

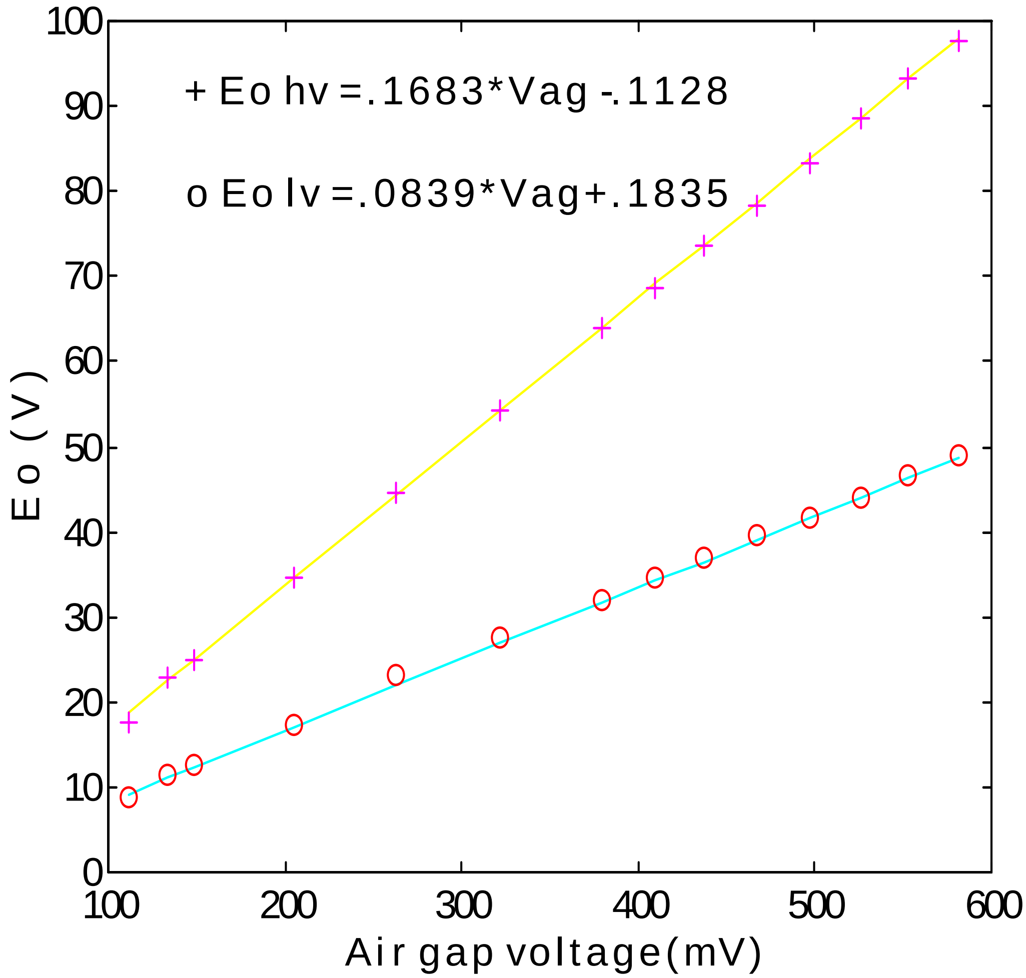

Figure 2.4. Measured line to neutral rms terminal generator voltage vs air gap voltage for no load condition for machine connected in high and low voltage stator connections

Hertz since most loads were designed to operate at or near that particular voltage. Although not shown explicitly on Figure 2.4, the machine is operating at 60 Hertz when the air gap voltage is approximately 520 mv. This operating point corresponds to a magnet voltage of about 88 volts line to neutral rms which is certainly low if one wanted to drive, for example an induction motor with it. As will be seen in subsequent chapters, shunt capacitors placed at the terminals of the PM machine boosts the terminal voltage, but a better method would be to design a PM machine having a magnet voltage of around 130 volts when operating at 60 Hertz. Incidentally, the lower voltage of this IPM machine is not an indication of poor design; rather, it is an indication that it was designed to be used as a motor.

The empirical relationship between the air gap voltage (Vag) and the magnet voltage on a per phase rms basis (E_o) was found to be

$$E_o = 0.1683 * V_{ag} - 0.1128 \quad . \tag{2.30}$$

With this relationship established, the magnet voltage (and thus the magnet flux) could be approximated under load conditions by measuring the air gap voltage at each operating condition. While this method is not entirely accurate since, under load conditions, the voltage across the search coil is also affected by the current flowing in the mutual inductances of the d and q windings of the stator, the approximation seems reasonable and, absent the use of finite element analysis, there is little other option but to use this method if the d axis inductance and the magnet flux terms are to both remain as functions of the operating conditions.

After the magnet flux term has been determined (by $\lambda_e = E_o / \omega$), then L_{ds} can be found by

$$L_{ds} = \frac{V_{qs} - r_s I_{qs} - \omega \lambda_e}{\omega I_{ds}} \quad . \tag{2.31}$$

The plots of L_{qs}, L_{ds}, and λ_e are given in Figures 2.5-2.8. The empirical relationships of the parameters as a function of peak stator current I_s are given as

$$\begin{aligned} \ln\left(\frac{1}{L_{qs}}\right) &= -.0013\, I_s^4 + .0293\, I_s^3 - .2303\, I_s^2 + .8684\, I_s + .790 \\ \ln\left(\frac{1}{L_{ds}}\right) &= -.0011\, I_s^4 + .0251\, I_s^3 - .210\, I_s^2 + .9096\, I_s + 1.505 \\ \lambda_e &= .0002\, I_s^3 - .0041\, I_s^2 + .0208\, I_s + .1863 \quad . \end{aligned} \tag{2.32}$$

In a conventional synchronous machine with a field winding, the d axis inductance is larger than the q axis inductance; however, comparing the magnitudes of L_{qs} and L_{ds} in Figures 2.5 and 2.6, it can be seen that L_{qs} is larger. This phenomenon, called inverse saliency, is caused by the magnet depth appearing as basically an air gap in the d-axis. As the machine is loaded, it can also be seen that the magnet voltage E_o first increases and then decreases. The initial increase in magnet voltage is due to the fact that, at very light loads, the bridge becomes highly saturated and much of the magnet flux flows through it and does not contribute towards a useful air gap voltage.

It was mentioned earlier in this section that the reason the parameters were made functions of the peak stator current and not the total flux linkage was that the when the

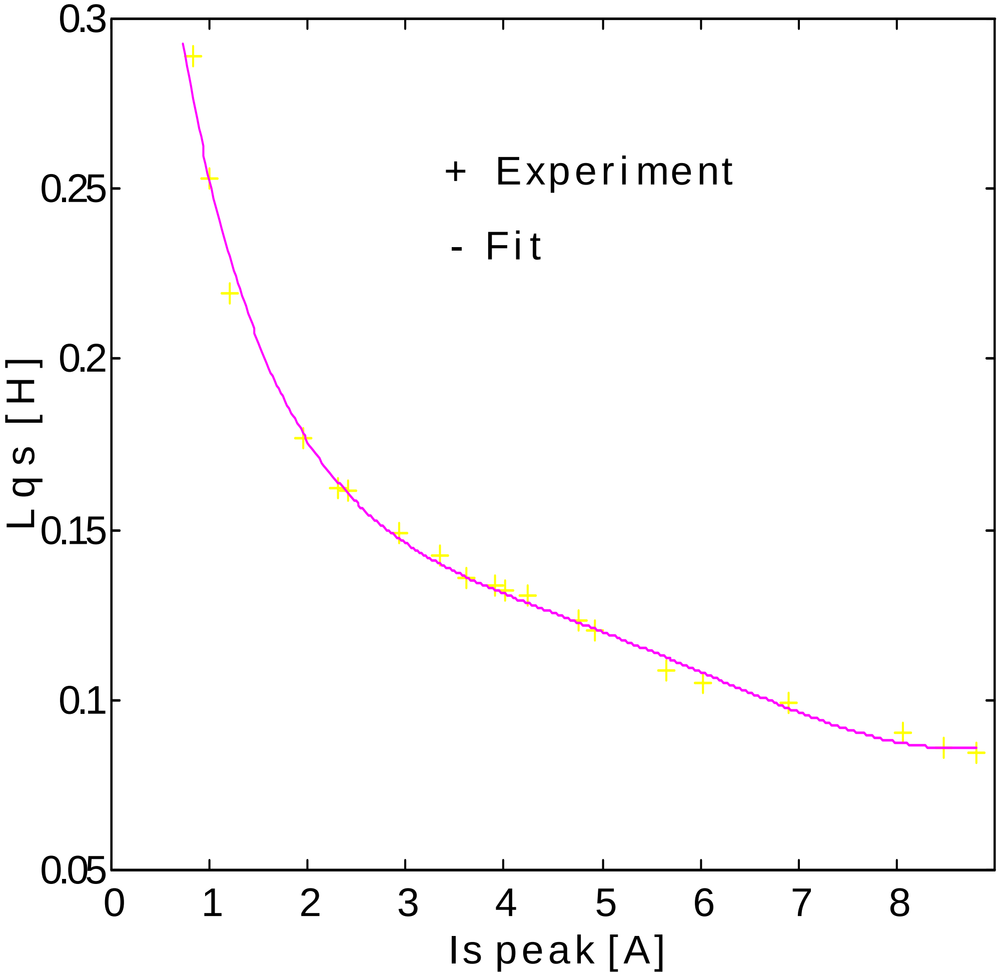

Figure 2.5. Measured values of q axis inductance vs peak stator current

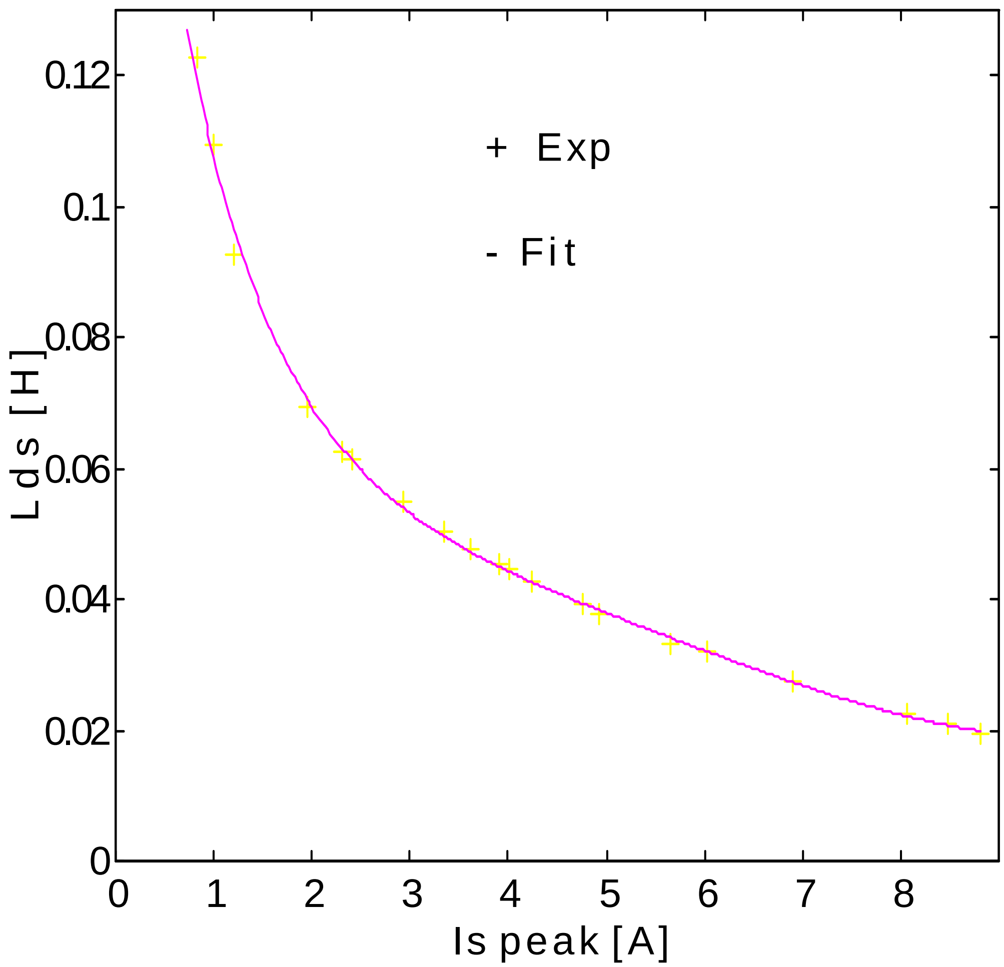

Figure 2.6. Measured d axis inductance vs peak stator current

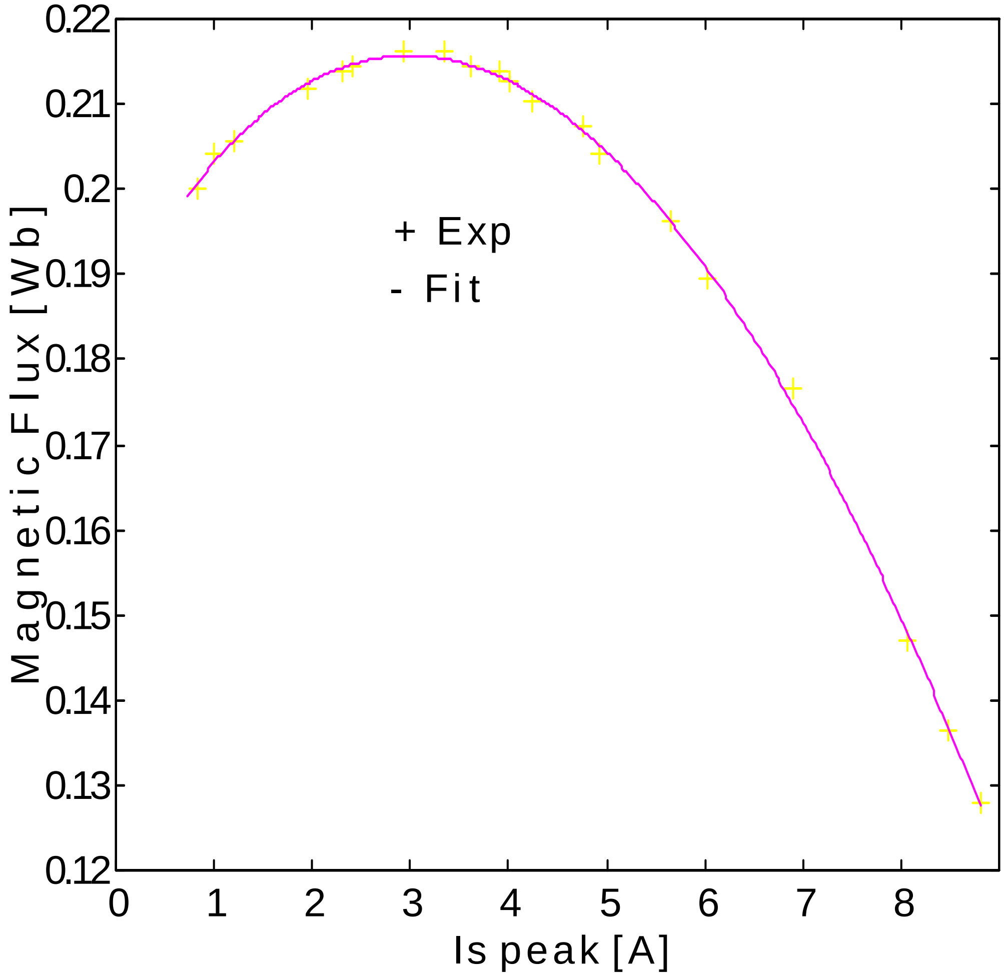

Figure 2.7. Measured magnetic flux vs peak stator current

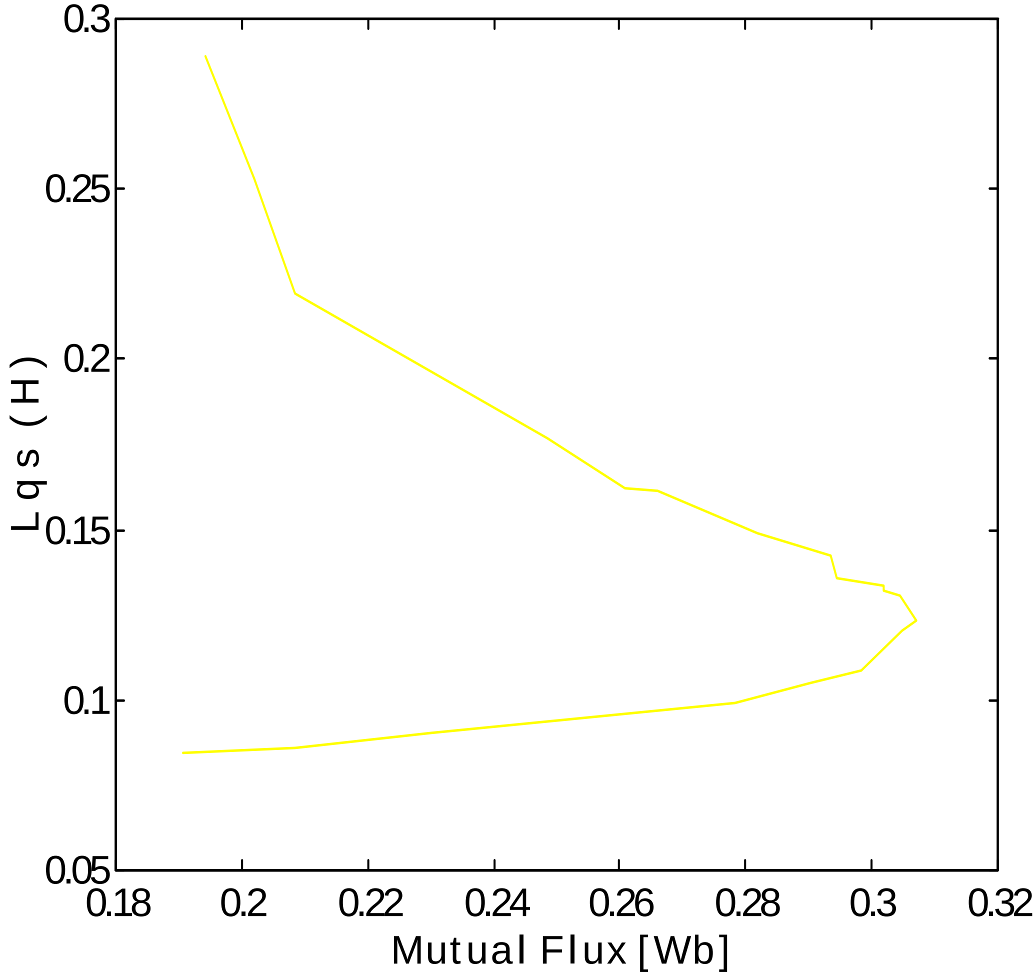

Figure 2.8. Measured q axis inductance vs peak mutual flux

parameters were plotted as a function of the total mutual flux linkage λ_{mm}, there were regions in which two possible solutions exist. An example of this is shown in Figure 2.8 where the q axis inductance is plotted vs the mutual flux linkage. It can be seen from the figure why determining L_{qs} from a given value of λ_{mm} would be difficult. For example, if λ_{mm} was given as 0.28 Wb, then L_{qs} could either be 0.1 or 0.15 H. The same problem was present for the parameters L_{ds} and λ_e as well.