CHAPTER 5

PWM SCHEMES IN FOUR- LEGGED CONVERTERS

5.1 Introduction

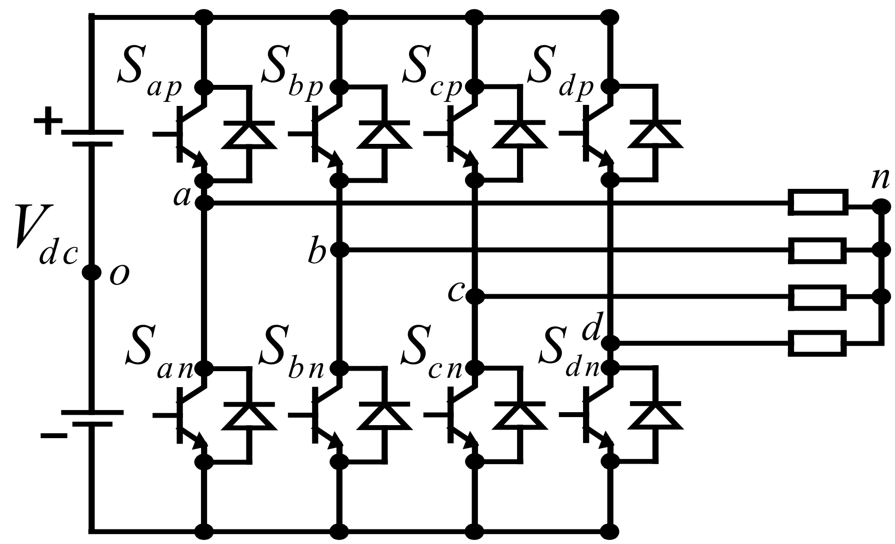


Figure 5.1 Circuit topology of four-leg DC/AC inverter.

In many commercial and industrial applications, power distribution is through a three-phase four-wire system. Such a system has some implicit problems. If the loads applied are nonlinear single phase or three phase or unbalanced, neutral currents containing both the fundamental and harmonics flow. This neutral current is potentially damaging to both the neutral conductor and the transformer to which it is connected. Limitations of the stand-alone three-phase four wire power supplies are compensated by four legged converters, which can deal with the nonlinearities and the load unbalance [5.2]. Four-leg converters are finding relevance in active power filters and fault-tolerant three-phase rectifiers with capability for load balancing and distortion mitigation [5.1-5.5] . For three phase, four-wire electric distribution systems, four-leg converters are now used in distributed generator systems such as micro-turbines to provide three-phase outputs with neutral connections [5.4]. The control of this converter is to ensure voltage and current regulation or power quality improvement. This control is achieved through the use of either carrier-based pulse-width modulation or space vector inverter control schemes. The space vector (3-D SVM) scheme for four-leg converter structure has been detailed experimentally demonstrated in [5.7]. This control schemes demands selection of the switching sequence by setting or clamping the device to the DC rail for an effective period of 120 degrees. However for 3-D SVM there are many possibilities for deciding the switching sequence. The disadvantage in this sequencing is that, depending on the application the switching sequence has to be programmed manually. This limitation is overcome in carrier based discontinuous PWM schemes for four legged inverters. By changing a set of parameters, derived from the switching table, any switching sequences are possible. Thus the scheme becomes an online adaptive one to meet the load requirements. The load requirements are in terms of harmonic distortion, switching losses, continuous or discontinuous mode of operation.

Fig. 5.1 shows the circuit topology of the four-leg voltage source DC/AC inverter in which the fourth leg, in general is connected through impedance to the neutral of the three-phase load, which could be, unbalanced or/and nonlinear. The turn-on and turn-off sequences of a switching transistor are represented by an existence function, which has a value of unity when it is turned on and becomes zero when it is off. In general, an existence function of a two-level converter is represented by S_{ij}, i = a,b,c, and d and j = p, n where i represents the load phase to which the device is connected, and j signifies top (p) and bottom (n) device of the inverter leg. Hence, S_{ap}, S_{an} which take values of zero or unity, are respectively the existence functions of the top device and bottom device of the inverter leg which is connected to phase 'a' load [A.3-5.12]. The load voltage equations expressed in terms of the existence functions and input DC voltage V_d are given as:

$$0.5V_d(S_{ap} - S_{dp}) = V_{an} + V_{no}$$

$$0.5V_d(S_{bp} - S_{dp}) = V_{bn} + V_{no}$$

$$0.5V_d(S_{cp} - S_{dp}) = V_{cn} + V_{no}$$

$$0.5V_d(S_{dp} - S_{dp}) = V_{dn} + V_{no} \tag{5.1}$$

5.2 Continuous Modulation in four legged inverter

In equations in (5.1), V_{an},$V_{bn,}$$V_{cn}$ are the desired phase voltages of the load and the phase voltage of the neutral impedance connected to the fourth leg is V_{dn}. The voltage between a reference 'o' of the inverter and the neutral of the load is denoted by $V_{no.}$ In order to prevent short-circuiting the DC source and thereby not violate the Kirchoff's voltage law, S_{ip} and S_{in} cannot be turned on at the same time. Hence, Kirchoff's law constraints the existence function such that $S_{ip} + S_{in}=1$. After an algebraic manipulation, with due considerations given to the constraints imposed by the voltage Kirchoff's law, equations in (5.1) reduce to:

$$V_d(S_{ap} - S_{dp}) = V_{an} - V_{dn}$$

$$V_d(S_{bp} - S_{dp}) = V_{bn} - V_{dn}$$

$$V_d(S_{cp} - S_{dp}) = V_{cn} - V_{dn} \tag{5.2}$$

It is desired to determine the expressions for the four $\mathbf{S_{ij}}$ from equations in (5.2) given the phase voltages V_{an} , $V_{bn,}$ $V_{cn,}$ V_{dn}. Since there are three linear independent equations to be solved to determine expressions for four unknown existence functions, these equations are under-determined. In view of this indeterminacy, there are infinite number of solutions which are obtained by various optimizing performance functions defined in terms of the existence functions. From (5.2) we have

$$[A].[M]=[Y] \tag{5.3}$$

Where,

$$A=\begin{bmatrix} V_d & 0 & 0 & -V_d \\ 0 & V_d & 0 & -V_d \\ 0 & 0 & V_d & -V_d \end{bmatrix} \quad Y=\begin{bmatrix} V_{an}-V_{dn} \\ V_{bn}-V_{dn} \\ V_{cn}-V_{dn} \end{bmatrix} \quad M=\begin{bmatrix} M_{ap} \\ M_{bp} \\ M_{cp} \\ M_{dp} \end{bmatrix}$$

For a set of linear indeterminate equations expressed as **AX = Y**, a solution which minimizes the sum of squares of the variable **X** is obtained using the Moore-Penrose inverse [A.3]. The solution is given as $\mathbf{X}= \mathbf{RA^T((ARA^T)^{-1})Y}$. where,

$$\mathbf{R}=\begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix}$$

The solutions for the minimization of the sum of the squares of the four existence functions (equivalently, this is the maximization of the inverter output-input voltage gain), i.e. $S_{ap}{}^2 + S_{bp}{}^2 + S_{cp}{}^2 + S_{dp}{}^2$ subject to the constraints in equations in (2) are given as [12] :

If we solve the above equations we get the equations for the modulation signals:

$$m_a=\frac{k_1(k_2k_3V_{an}-k_2k_3V_{dn}+k_2k_4V_{an}-k_2k_4V_{dn}+k_4k_3V_{an}-k_4k_3V_{dn}-k_4k_3V_{bn}+k_4k_3V_{dn}-k_2k_4V_{cn}+k_2k_4V_{dn}}{V_d(k_2k_3k_1+k_2k_4k_1+k_4k_3k_1+k_2k_3k_4)}$$

$$m_b = \frac{k_2(-k_4k_3V_{an} + k_4k_3V_{dn} + k_1k_3V_{bn} - k_1k_3V_{dn} + k_4k_1V_{bn} - k_4k_1V_{dn} + k_4k_3V_{bn} - k_4k_3V_{dn} - k_1k_4V_{cn} + k_1k_4V_{dn}}{V_d(k_2k_3k_1 + k_2k_4k_1 + k_4k_3k_1 + k_2k_3k_4)}$$

$$m_c = \frac{k_3(-k_2k_4V_{an} + k_4k_2V_{dn} - k_1k_4V_{bn} + k_1k_4V_{bn} + k_1k_2V_{cn} - k_2k_1V_{dn} + k_4k_1V_{cn} - k_4k_1V_{dn} + k_2k_4V_{cn} - k_2k_4V_{dn}}{V_d(k_2k_3k_1 + k_2k_4k_1 + k_4k_3k_1 + k_2k_3k_4)}$$

$$m_d = \frac{-k_4(k_2k_3V_{an} - k_3k_2V_{dn} + k_1k_3V_{bn} - k_1k_3V_{dn} + k_2k_1V_{cn} - k_2k_1V_{dn})}{V_d(k_2k_3k_1 + k_2k_4k_1 + k_4k_3k_1 + k_2k_3k_4)}$$

When $K_1 = K_2 = K_3 = K_4 = 1$, we get

$$m_a = \frac{3}{4V_d}(V_a - V_{dn}) - \frac{1}{4V_d}(V_b - V_{dn}) - \frac{1}{4V_d}(V_{cn} - V_{dn})$$

$$m_b = \frac{-1}{4V_d}(V_{an} - V_{dn}) + \frac{3}{4V_d}(V_{bn} - V_{dn}) - \frac{1}{4V_d}(V_{cn} - V_{dn})$$

$$m_c = \frac{-1}{4V_d}(V_{an} - V_{dn}) - \frac{1}{4V_d}(V_{bn} - V_{dn}) + \frac{3}{4V_d}(V_{cn} - V_{dn})$$

$$m_d = \frac{-1}{4V_d}(V_{an} - V_{dn}) - \frac{1}{4V_d}(V_{bn} - V_{dn}) - \frac{1}{4V_d}(V_{cn} - V_{dn}) \qquad (5.4)$$

Simplifying we have,

$$M_{ap} = \frac{1}{4}(3V_{ann} - V_{bnn} - V_{cnn} - V_{dnn})$$

$$M_{bp} = \frac{1}{4}(-V_{ann} + 3V_{bnn} - V_{cnn} - V_{dnn})$$

$$M_{cp} = \frac{1}{4}(-V_{ann} - V_{bnn} + 3V_{cnn} - V_{dnn})$$

$$M_{dp} = \frac{1}{4}(-V_{ann} - V_{bnn} - V_{cnn} + 3V_{dnn}) \qquad (5.5)$$

$V_{ann} = V_{an}/ V_d$

$V_{bnn} = V_{bn}/ V_d$

$V_{cnn} = V_{cn}/ V_d$

$V_{dnn} = V_{dn}/ V_d$

M_{ip} are the continuous PWM modulation signals for the top devices of the four inverter legs. These signals are compared with a high frequency triangle carrier waveform (ranging from + 1 to –1) to generate the PWM switching pulses for the base drives of the switching devices.

5.2.1. Experimental results

The proposed modulation schemes are practically implemented by means of a floating-point 40-MHz TMS320LF2407A DSP to synthesize three-phase unbalanced phase voltages. Fig. 5.2 shows the experimentally generated balanced reference three-phase voltages using the continuous modulation signals set forth in (5.5). The modulation signals for the top four devices are also displayed showing that for balanced phase voltages, the modulation signal for the fourth leg is zero. Fig. 5.3 gives the experimental waveforms when a three-phase unbalanced voltage set with the magnitude of phase ‘a’ voltage reduced by 20% is synthesized. The maximum current and voltage rating of the converter used was 14A and 350VDC. The DC voltage applied was 80V and the output load comprised of an inductance of 4mH and a filter capacitor of 30uF in each phase. The Figures. 5.2 and 5.3 largely confirm the correctness of the proposed continuous modulation scheme [5.12].

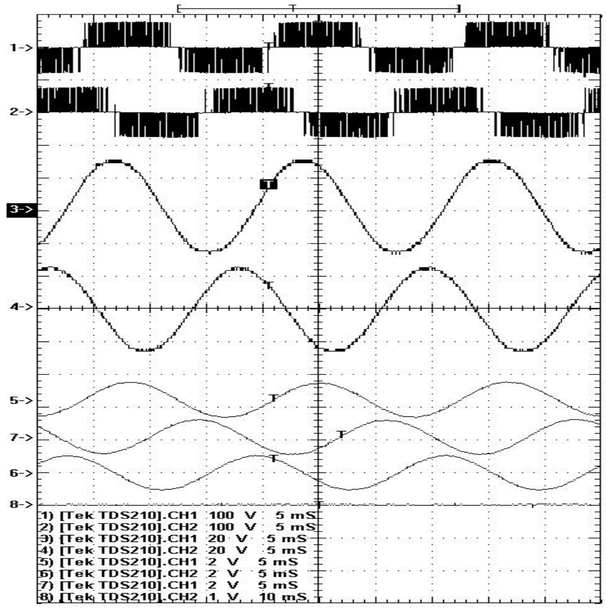


Figure 5.2 Experimental results: Generation of balanced three-phase voltages using continuous modulation scheme. V_{an} = 25 cos (377t), V_{bn} =25 cos (377t - 2π/3), V_{cn} = 25 cos (377t + 2π/3), V_d = 80V. (1-2) phase 'a' and 'b' voltages, (3-4) filtered phase 'a' and 'b' voltages, (5-8) phase modulating signals.

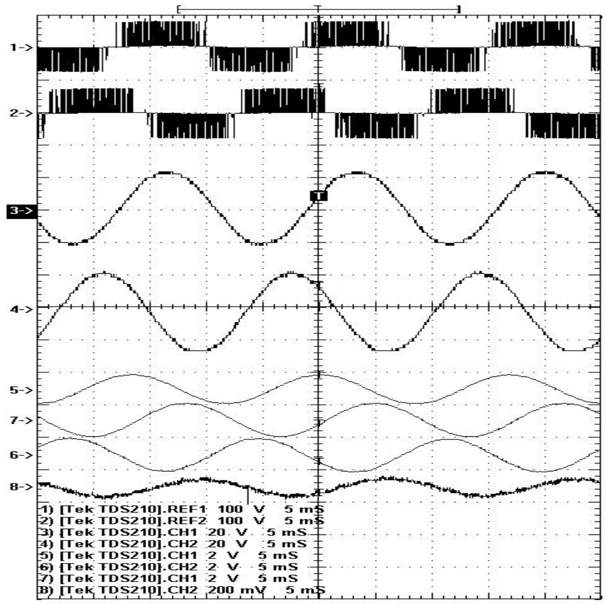


Figure 5.3 Experimental results: Generation of unbalanced three-phase voltages. Unbalanced voltages are V_{an} = 20 cos (377t), V_{bn} =25 cos (377t - 2π/3), V_{cn} = 25 cos (377t + 2π/3) V_d = 80V. (1-2) phase 'a' and 'b' voltages, (3-4) filtered phase 'a' and 'b' voltages, (5-8) phase modulating signals.

5.3 Carrier based discontinuous scheme using space vector modulation approach

This chapter makes a novel contribution to the development of the carrier-based generalized discontinuous modulation scheme for the four-leg DC/AC inverters. Through intensive analyses and experimental results, the modulation possibilities opened up are discussed and the relations between the new carrier-based technique and the 3-D space vector modulation scheme in [5.7] are explicated.

The 16 feasible switching modes of the four-leg inverter of Figure 5.1 are given in Table 5.1 [5.2]. The stationary reference frame qdo voltages of the switching modes are expressed in the complex variable form as ($a = e^{j\beta}$, $\beta = 120°$):

$$V_{qds} = \frac{2}{3}(V_{an} + aV_{bn} + a^2V_{cn}), \quad \text{and} \quad V_o = \frac{1}{3}(V_{an} + V_{bn} + V_{cn}) \tag{5.6}$$

Using the phase to reference voltages V_{ao}, V_{bo}, V_{co} and V_{do} for each switching mode, the components of the stationary reference frame V_{qdos} expressed in terms of the switching functions are given as:

$$V_{qs} = \frac{V_d}{6}(2S_{ap} - S_{bp} - S_{cp} - 2S_{an} + S_{bn} + S_{cn})$$

$$V_{qs} = \frac{V_d}{3}(2S_{ap} - S_{bp} - S_{cp}) \tag{5.7}$$

$$V_{ds} = \frac{V_d}{\sqrt{3}}(S_{cp} - S_{bp} - S_{cn} + S_{bn})$$

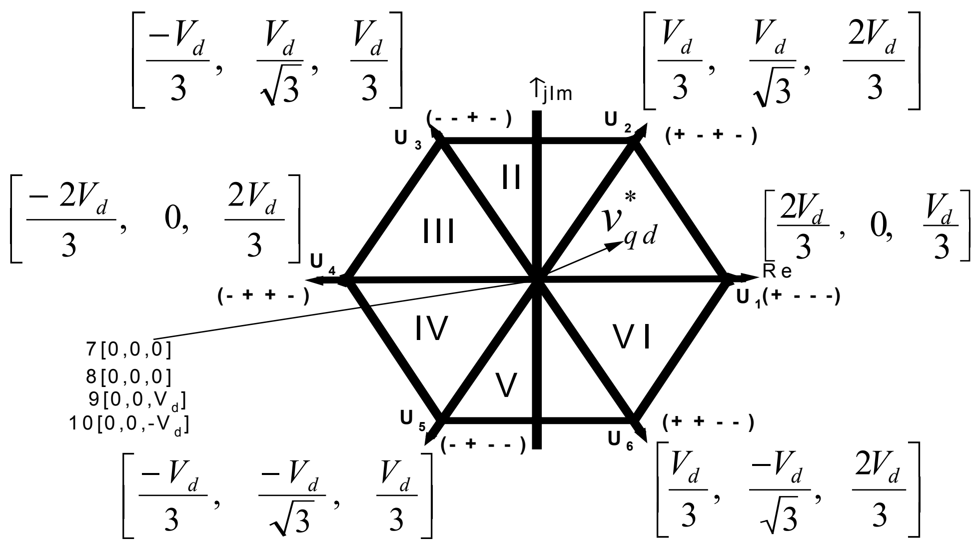
$$V_{ds} = \frac{V_d}{\sqrt{3}}(S_{cp} - S_{bp}) \tag{5.8}$$

$$V_{os} = \frac{V_d}{6}(S_{ap} + S_{bp} + S_{cp} - S_{an} - S_{bn} - S_{cn} - 3S_{dp} + 3S_{dn}) + V_{dn}$$

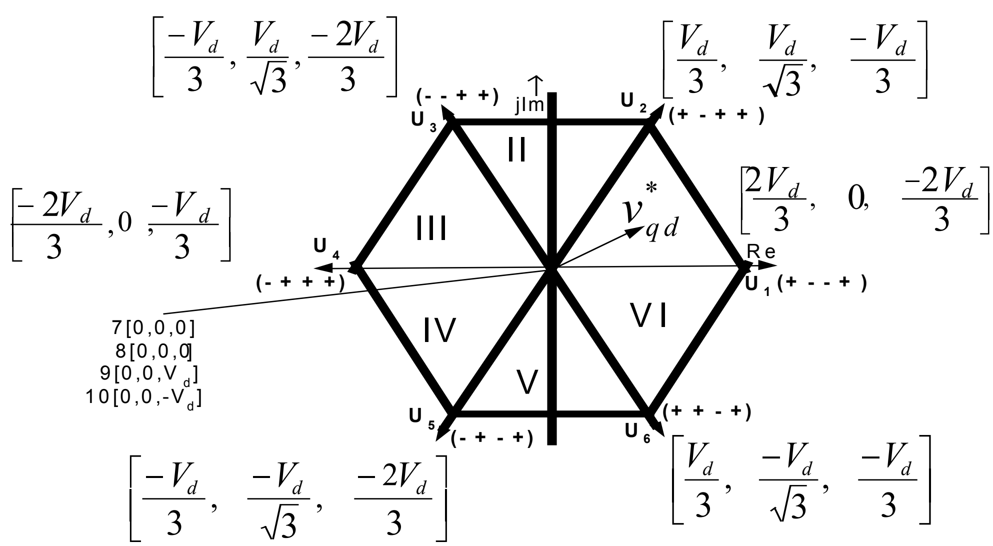
$$= \frac{V_d}{3}(S_{ap} + S_{bp} + S_{cp} - 3S_{dp}) + V_{dn} \tag{5.9}$$

Table 5.1 Switching modes and qdo voltages

S_{ap}	S_{bp}	S_{cp}	S_{dp}	$3V_q^*$	$\sqrt{3}V_d^*$	$3V_o^*$	Mode
0	0	0	0	0	0	0	7
0	0	0	1	0	0	$-3V_d$	10
0	0	1	0	$-V_d$	$-V_d$	$-V_d$	3a
0	0	1	1	$-V_d$	$-V_d$	$-2V_d$	3b
0	1	0	0	$-V_d$	$-V_d$	$-V_d$	5a
0	1	0	1	$-V_d$	$-V_d$	$-2V_d$	5b
0	1	1	0	$-2V_d$	0	$-2V_d$	4a
0	1	1	1	$-2V_d$	0	$-V_d$	4b
1	0	0	0	$2V_d$	0	V_d	1a
1	0	0	1	$2V_d$	0	$-2V_d$	1b
1	0	1	0	V_d	V_d	$2V_d$	2a
1	0	1	1	V_d	V_d	$-V_d$	2b
1	1	0	0	V_d	$-V_d$	$2V_d$	6a
1	1	0	1	V_d	$-V_d$	$-V_d$	6b
1	1	1	0	0	0	$3V_d$	9
1	1	1	1	0	0	0	8



(a)



(b)

Figure. 5.4 Space vector of switching modes. (a) Positive zero sequence voltages, (b) negative zero sequence voltages.

It is evident from Table 5.1 that the 16 switching modes can be divided into three broad divisions.

I. Set I comprising of modes (1a-6a) in which all the zero sequence voltages are positive as shown in Figure 5.4 (a)

II. Set II comprising of modes (1b-6b) in which all the zero sequence voltages are negative as shown in Figure 5.4 (b) and

III. Set III comprising the null states (7,8,9,10) common to both the figures.

Modes 7 and 8 are two null states with zero qdo voltages while modes 9 and 10 are modes with zero qd voltages having zero sequence voltages of equal but opposite sign. A space vector methodology based on the partitioning of modes as shown in Figure. 5.4 is set forth in which the null states 7,8,9,10 are common to both. It is mandatory to consider the zero sequence voltages for the switching modes must be included in the calculations since the inverters are used in systems with unbalanced and nonlinear loads. In classical space vector technique, a reference voltage V_{qd}^{*} located within the six sectors of the complex space vector in Figure. 5.4 is approximated instantaneously by time-averaging of four vectors comprising of two adjacent active switching modes and the two null modes 0,7 over the PWM sampling period T_s. The reference voltage is approximated by time-averaging six switching modes comprising of two active modes which are adjacent to the reference V_{qdo}^{*}, and the four null voltage modes 7,8,9,10. For the synthesis of a voltage reference set, there are four possible two active switching modes, which can be used. These are obtained by using (a) two active adjacent states in the positive sequence set, fig. 2(a); (b) two adjacent active states in the negative sequence set, (c) selecting an adjacent active mode from the negative and the other adjacent active set from the positive sequence set and vice visa. The synthesis of a reference voltage in sector I can be explained using Table I and Fig 5.4.

Non-overlap of switching sequences

a. If the switching modes from the positive sequence itself are to be used then, 1(a), 2(a),7,8,9,10 are required

b. If the switching modes from the negative sequence itself are to be used then, 1(b), 2(b),7,8,9,10 are required

Overlap of switching sequences

If the switching modes from the positive sequence and negative sequence as well are to be used then, 1(a), 2(b),7,8,9,10 and 1(b),2(a),7,8,9,10 can be used.

The normalized times the active modes (V_{qdoa} , V_{qdob}) are used are t_a and t_b respectively, t_d is the combined normalized time modes 9 and 10 are applied and the combined times modes 7 and 8 are utilized is t_c . If mode 7 is applied for $(1-\kappa)t_c$, mode 8 for κt_c , mode 9 for γt_d and mode 10 for $(1-\gamma)\, t_d$, and if $T_a + T_b + T_c + T_d = T_s$ is the total switching period then, normalizing these times we have,

$t_d = 1-(t_a + t_b + t_c)$ where $t_c = t_7 + t_8$ and $t_d = t_9 + t_{10}$

Let $t_8 = \kappa t_c$ and $t_9 = \gamma t_d$ hence $t_7 = (1-\kappa)t_c$ and $t_{10} = (1-\gamma)\, t_d,$ where , $0 \le \kappa \le 1$, $0 \le \gamma \le 1$

(5.10)

So we have the expression for reference voltage vectors given as:

$$V_{qdo}^{*} = V_{qdo}t_a + V_{qdob}t_b + V_{qdo7}(1-\kappa)t_c + V_{qdo8}\kappa t_c + V_{qdo9}\gamma t_d + V_{qdo10}(1-\gamma)t_d \qquad (5.11)$$

$$V_o^{*} = V_{oa}t_a + V_{ob}t_b + V_{07}(1-\kappa)t_c + V_{o8}\kappa t_c + V_{09}\gamma t_d + V_{o10}(1-\gamma)t_d \qquad (5.12)$$

Therefore $t_d = (V_o^{*} - V_{oa}t_a - V_{ob}t_b)/(V_d(2\gamma - 1)$ (5.13)

It is observed that both V_{qd7} $V_{qd8,}$ V_{qd9} and V_{qd10} do not influence the values of t_a and t_b . It is obvious that in case of the four legged inverter the active times has to be the same as those of the three phase inverter because in this topology, the only difference should lie in processing the zero sequence component of voltage/current through the neutral or the fourth leg. The times t_a and t_b are given in Table 5.2 for voltage references in the six sectors for positive and negative sequence which is the same as those derived in Chapter 3 Table 3.4.

Thus times t_a and t_b as listed in Table 5.2 and the Null states are common to any switching mode. The primary difference lies in utilization of the zero sequence voltage for the cases considered above and hence imposing various conditions of t_d and t_c.

Table 5.2 Device switching times expressed in term of qd reference voltage

Sectors	I	II	III	IV	V	VI
t_a	$\frac{0.5}{V_d}(3V_{qq}-\sqrt{3}V_{dd})$	$\frac{0.5}{V_d}(3V_{qq}+\sqrt{3}V_{dd})$	$\frac{\sqrt{3}V_{dd}}{V_d}$	$\frac{0.5}{V_d}(-3V_{qq}+\sqrt{3}V_{dd})$	$\frac{0.5}{V_d}(-3V_{qq}-\sqrt{3}V_{dd})$	$\frac{-\sqrt{3}V_{dd}}{V_d}$
t_b	$\frac{\sqrt{3}V_{dd}}{V_d}$	$\frac{0.5}{V_d}(-3V_{qq}+\sqrt{3}V_{dd})$	$\frac{-0.5}{V_d}(3V_{qq}+\sqrt{3}V_{dd})$	$\frac{-\sqrt{3}V_{dd}}{V_d}$	$\frac{0.5}{V_d}(3V_{qq}-\sqrt{3}V_{dd})$	$\frac{0.5}{V_d}(3V_{qq}+\sqrt{3}V_{dd})$

To express the average times obtained from Table 5.2 in terms of a-b-c voltages Table 5.3 we use the inverse transformation equations given by:

$$f_a = f_q + f_o ; f_b = \frac{-f_q}{2} - \frac{\sqrt{3} f_d}{2} + f_o ; f_c = \frac{-f_q}{2} + \frac{\sqrt{3} f_d}{2} + f_o$$

Consider:

$$f_a - f_b = f_{ab} = f_q + f_o + \frac{f_q}{2} + \frac{\sqrt{3} f_d}{2} + f_o = 0.5(3 f_q - \sqrt{3} f_d) / V_d$$

$$f_a - f_c = f_{ac} = f_a + f_o + \frac{f_q}{2} + \frac{\sqrt{3} f_d}{2} + f_o = 0.5(3 f_q + \sqrt{3} f_d) / V_d$$

$$f_b - f_c = f_{bc} = \frac{-f_q}{2} - \sqrt{3} \frac{f_d}{2} + f_o + \frac{f_q}{2} - \frac{\sqrt{3} f_d}{2} - f_o = \sqrt{3} f_d / V_d \tag{5.14}$$

From (5.14) we can express the dwell times of the active devices as $V_d t_a$ and $V_d t_b$ as given in Table 5.3

Table 5.3 Dwell times of the active devices.

Sect	**I**	**II**	**III**	**IV**	**V**	**VI**
$V_d t_a$	V_{ac}	V_{ab}	V_{cb}	V_{ca}	V_{ba}	V_{bc}
$V_d t_b$	V_{cb}	V_{ca}	V_{ba}	V_{bc}	V_{ac}	V_{ab}

5.3.1 Conditions for positivity of dwell times - td and tc

From eqn (5.13) we have $t_d = \dfrac{(V^*{}_0 - V_{aa}t_a - V_{ob}t_b)}{V_d(2\gamma - 1)}$ value of t_d depends on the way in which the zero sequence voltages V_{oa} and V_{ob} are selected. The value of γ is selected such that t_d is always positive.

Thus if $t_d >= 0$ $\gamma = 1$ else $\gamma = 0$.

These zero sequence voltage can be selected from Table 5.1 in four ways.

i. Select V_{oap} & V_{obp} directly from the positive sequence
ii. Select V_{oan} & V_{obn} directly from the negative sequence.
iii. Select V_{oap} from the positive and V_{obn} from the negative sequence
iv. Select V_{obp} from the positive and V_{oan} from the negative sequence

Hence there will be four possible positive solutions for t_d in every sector depending on the selection of the zero sequence voltages as shown in Tables 5.4(a) through 5.4(d).

Table 5.4(a) V_{oap} and V_{obp}

Sect	I	II	III	IV	V	VI
$3V_{oa}$	V_d	$2V_d$	V_d	$2V_d$	V_d	$2V_d$
$3V_{ob}$	$2V_d$	V_d	$2V_d$	V_d	$2V_d$	V_d

Table 5.4(b) V_{oan} and V_{obn}

Sect	I	II	III	IV	V	VI
$3V_{oa}$	$-2V_d$	$-V_d$	$-2V_d$	$-V_d$	$-2V_d$	$-V_d$
$3V_{ob}$	$-V_d$	$-2V_d$	$-V_d$	$-2V_d$	$-V_d$	$-2V_d$

Table 5.4(c) V_{oap} and V_{obn}

Sect	I	II	III	IV	V	VI
$3V_{oap}$	V_d	$2V_d$	V_d	$2V_d$	V_d	$2V_d$
$3V_{obn}$	$-V_d$	$-2V_d$	$-V_d$	$-2V_d$	$-V_d$	$-2V_d$

Table 5.4 (d) V_{obp} and V_{oan}

Sect	I	II	III	IV	V	VI
$3V_{obp}$	$2V_d$	V_d	$2V_d$	V_d	$2V_d$	V_d
$3V_{oan}$	$-2V_d$	$-V_d$	$-2V_d$	$-V_d$	$-2V_d$	$-V_d$

Now using equation (5.13) and (5.6) and Table 5.4(a), for Sector I

We have, condition of t_d in positive sequence expressed as:

$$t_d(p) = \frac{1}{3}(V_{an} + V_{bn} + V_{cn}) - V_{dn} - \frac{\frac{1}{3}(V_d t_a - 2V_d t_b)}{V_s(2\gamma - 1)}$$

From table 5.3 for sector I we have, $V_d t_a = V_{ac}$ and $V_d t_b = V_{cb}$ hence,

$$t_d(p) = (\frac{1}{3}(V_{an} + V_{bn} + V_{cn}) - V_{dn} - \frac{\frac{1}{3}(V_{ac} - 2V_{cb})}{V_d(2\gamma - 1)}$$

$$t_d(p) = \frac{(\frac{1}{3}(V_{an} + V_{bn} + V_{cn} - V_{an} + V_{cn} - 2V_{cn} + 2V_{cn} + 2V_{bn} - 3V_{dn})}{V_d(2\gamma - 1)}$$

$$t_d(p) = \frac{(\frac{1}{3}(3V_{bn} - 3V_{dn})}{V_d(2\gamma - 1)}$$

$$t_d(p) = \frac{(V_{bn} - V_{dn})}{V_d(2\gamma - 1)} \tag{5.15}$$

Similarly for negative sequence in Sector I, using eqn (5.13) and (5.6) and Table 5.4(b),

$$t_d(n) = \frac{(V_{bn} - V_{dn})}{V_d(2\gamma - 1)} \tag{5.16}$$

Now we define two values of t_d using tables 5.4(c) and 5.4(d) as $t_{d(I)}$ and $t_{d(II)}$ respectively.

Thus using eqn(5.13) and (5.6) and Table 5.4(c), for sector I we have,

$$t_d(I) = \frac{(V_{cn} - V_{dn})}{V_d(2\gamma - 1)} \tag{5.17}$$

and using eqn(5.13) and (5.6) and Table 5.4(d), for sector I we have,

$$t_d(II) = \frac{(V_{an} + V_{bn} - V_{cn} - V_{dn})}{V_d(2\gamma - 1)} \tag{5.18}$$

Table 5.5 Value of td with all the four cases.

Sector	I	II	III	IV	V	VI
$t_d(p)$	$V_{bdd}\chi$	$V_{bdd}\chi$	$V_{add}\chi$	$V_{add}\chi$	$V_{cdd}\chi$	$V_{cdd}\chi$
$t_d(n)$	$V_{cdd}\chi$	$V_{cdd}\chi$	$V_{cdd}\chi$	$V_{bdd}\chi$	$V_{bdd}\chi$	$V_{add}\chi$
$t_d(I)$	$V_{cdd}\chi$	$(V_{bdn} + V_{cdn} - V_{adn} - V_{ddn})\chi$	$V_{bdd}\chi$	$(V_{adn} + V_{bdn} - V_{cdn} - V_{ddn})\chi$	$V_{add}\chi$	$(V_{cdn} + V_{adn} - V_{bdn} - V_{ddn})\chi$
$t_d(II)$	$(V_{adn} + V_{bdn} - V_{cdn} - V_{ddn})\chi$	$V_{add}\chi$	$(V_{cdn} + V_{adn} - V_{bdn} - V_{ddn})\chi$	$V_{cdd}\chi$	$(V_{bdn} + V_{cdn} - V_{adn} - V_{ddn})\chi$	$V_{bdd}\chi$

$V_{idn} = \frac{V_{in}}{V_d}$ i=a,b,c and d , $\frac{(V_{kn} - V_{dn})}{V_d}$ k=a,b,c and $\chi = \frac{1}{2(2\gamma - 1)}$

Table 5.5 summarizes the values of dwell time t_d for all the possibilities through selection of the zero sequence voltages. This means that in Sector I for all the four cases the absolute value of the phase voltages, will decide the time t_d required to synthesize the reference signal. γ ensures t_d >0. Once the time t_d is decided, then from eqn 5.10 condition $t_c = 1 - (t_a + t_b + t_d) \geq 0$ $t_c = 0$ must be satisfied, i.e. 1- t_a - t_b - t_d >= 0.

Thus for Sector I,

$t_a + t_b = \frac{(V_a - V_c)}{V_d} + \frac{(V_c - V_b)}{V_d} = \frac{(V_{ab})}{V_d} = V_{abn}$ and

hence, $t_c = 1 - V_{abn} - t_d >= 0$ (5.19)

Where, $t_d(p) = \frac{(V_{bn} - V_{dn})}{V_d(2\gamma - 1)} or \quad t_d(n) = \frac{(V_{cn} - V_{dn})}{V_d(2\gamma - 1)} or$

$$t_d(I) = \frac{(V_{bn} - V_{dn})}{V_d(2\gamma - 1)} or \; t_d(II) = \frac{(V_{an} + V_{bn} - V_{cn} - V_{dn})}{V_d(2\gamma - 1)}$$

5.3.2 Algorithm for selection of t_d

It is mandatory that the dwell times should be greater than zero. For every possible condition of t_d there exists a corresponding condition for t_c . But if t_c turns out to be negative then, this particular case has to be dropped. In other words this t_c is to be made equal to zero. Depending upon the balanced or unbalanced reference voltages to be synthesized, out of the four possibilities of t_d there exist 16 possible conditions for t_c. i.e:

$t_{c(p)}>0$, $t_{c(n)}> 0$ $t_{c(I)}>0$, $t_{c(II)}>0$

$t_{c(p)}>0$, $t_{c(n)}> 0$ $t_{c(I)}>0$, $t_{c(II)} = 0$

$t_{c(p)}>0$, $t_{c(n)}>0$, $t_{c(II)}>0$, $t_{c(I)} = 0$

$t_{c(p)}>0$, $t_{c(I)}>0$, $t_{c(II)}>0$, $t_{c(n)} = 0$

$t_{c(n)}>0$, $t_{c(I)}>0$, $t_{c(II)}>0$, $t_{c(p)} = 0$

$t_{c(n)}>0$, $t_{c(p)}>0$, $t_{c(II)}>0$, $t_{c(I)} = 0$

$t_{c(n)}>0$, $t_{c(p)}>0$, $t_{c(II)} = 0$, $t_{c(I)} = 0$

$t_{c(I)}>0$, $t_{c(II)}>0$, $t_{c(p)} = 0$, $t_{c(n)} = 0$

$t_{c(n)}>0$, $t_{c(I)}>0$, $t_{c(II)} = 0$, $t_{c(p)} = 0$

$t_{c(n)}>0$, $t_{c(II)}>0$, $t_{c(I)} = 0$, $t_{c(p)} = 0$

$t_{c(p)}>0$, $t_{c(I)}>0$, $t_{c(II)} = 0$, $t_{c(n)} = 0$

$t_{c(p)}>0$, $t_{c(II)}>0$, $t_{c(n)} = 0$, $t_{c(I)} = 0$

$t_{c(p)}>0$, $t_{c(II)} = 0$, $t_{c(n)} = 0$, $t_{c(I)} = 0$

$t_{c(n)}>0$, $t_{c(II)} = 0$, $t_{c(p)} = 0$, $t_{c(I)} = 0$

$t_{c(I)}>0$, $t_{c(II)} = 0$, $t_{c(p)} = 0$, $t_{c(I)} = 0$

$t_{c(II)}>0$, $t_{c(p)} = 0$, $t_{c(n)} = 0$, $t_{c(I)} = 0$

Only those conditions of t_d, for corresponding t_c greater than zero can be used in synthesizing the reference voltage vector. In case of multiple conditions being satisfied then an additional criteria is introduced. Thus if all the four condition of t_c are satisfied, then all the four cases of t_d are valid hence either the maximum or minimum of these t_d is selected.

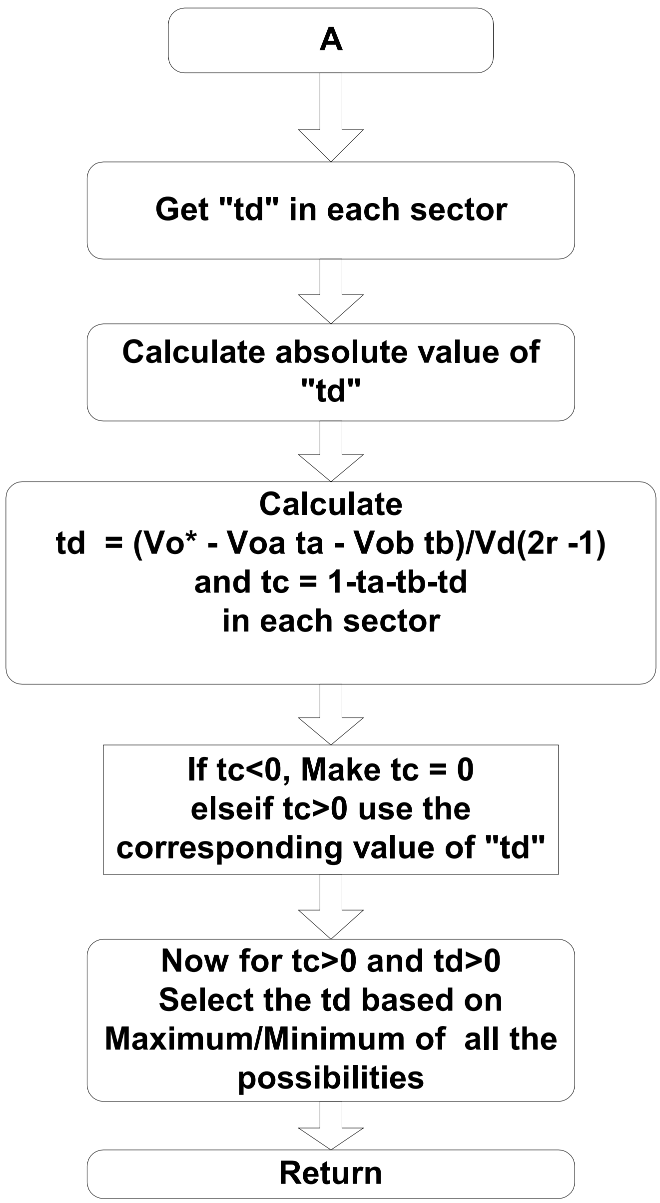


Figure 5.5 Flow chart for selection of condition of td

5.3.3 Existence function in four legged converter

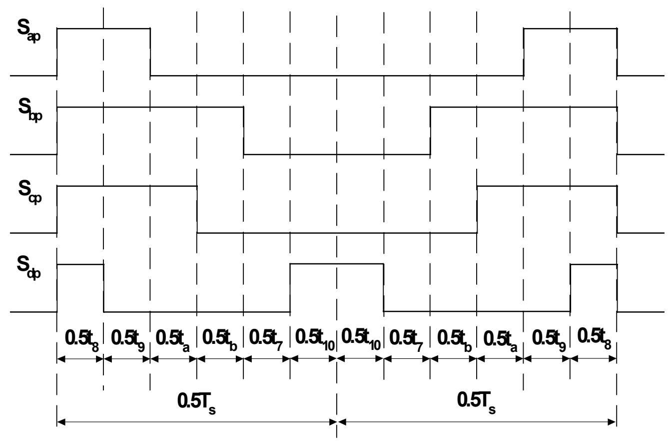


Figure 5.6 Existence function of the four leg converter in sector IV

The voltage equations expressed in terms of the modulation signals in (5.3) are facilitated by the Fourier series approximation of the existence functions, which are approximated as [A.2-A.3]:

$S_{ap} \cong Z_{ap} = 0.5\,(1 + M_{ap})$

$S_{bp} \cong Z_{bp} = 0.5\,(1 + M_{bp})$

$S_{cp} \cong Z_{cp} = 0.5\,(1 + M_{cp})$

$S_{dp} \cong Z_{dp} = 0.5\,(1 + M_{dp})$ (5.20)

Where, M_{ap} , M_{bp}, M_{cp}, M_{dp} which range between –1 and 1 (for the linear modulation range) are the carrier-based modulation waveforms comprising of fundamental frequency components. The approximate existence functions (Z_{ap}, Z_{bp}, Z_{cp} , Z_{dp}) which range between zero and unity can be used to generate actual existence functions by comparing them with a high frequency triangular waveform that ranges between unity and zero.

The symmetric switching sequence is adopted which is presumed to have low THD in view of the symmetry of the waveforms. For a reference voltage in sector IV in Figure .5.2(a), a period switching sequence is

$1111 \rightarrow 1110 \rightarrow 110 \rightarrow 0100 \rightarrow 0000 \rightarrow 0001 \rightarrow 0001 \rightarrow 0000 \rightarrow 0100 \rightarrow 0110 \rightarrow 1110 \rightarrow 1111$.

For a reference voltage in sector IV in Figure. 5.2(b),

$1110 \rightarrow 1111 \rightarrow 0111 \rightarrow 0101 \rightarrow 0001 \rightarrow 0000 \rightarrow 0000 \rightarrow 0001 \rightarrow 0101 \rightarrow 0111 \rightarrow 1111 \rightarrow 1110$

results.

The existence functions of the four top devices for realizing a reference voltage in sector IV in Fig. 5.4 (a) are shown in Fig. 5.6.

The expressions for the discontinuous modulation signals for the devices are determined by averaging their existence functions (such as those in Fig. 5.6) in each sector of Fig. 5.4. It is seen from Fig. 5.6 that the average of an existence function is equal to the sum of the normalized times each device is turned on to realize a reference voltage. Based on Tables 5.1 and 5.2 and (5.10), the total time each top device is turned on for the six sectors are determined and given in Table 5.6(a) and (b). The Table 5.6(b) corresponds to the normalized times of the switching devices turned on for the different cases of t_d that will be selected in synthesizing the reference voltage vector.

Table 5.6(a) Normalized times devices for the three top devices.

Sector	I	II	III	IV	V	VI
(Z_{ap})	$t_a + t_b + t_9 + t_8$	$t_a + t_9 + t_8$	$t_9 + t_8$	$t_9 + t_8$	$t_b + t_9 + t_8$	$t_a + t_b + t_9 + t_8$
(Z_{bp})	$t_9 + t_8$	$t_9 + t_8$	$t_b + t_9 + t_8$	$t_a + t_b + t_9 + t_8$	$t_a + t_b + t_9 + t_8$	$t_a + t_9 + t_8$
(Z_{cp})	$t_b + t_9 + t_8$	$t_a + t_b + t_9 + t_8$	$t_a + t_b + t_9 + t_8$	$t_a + t_9 + t_8$	$t_9 + t_8$	$t_9 + t_8$

Table 5.6(b) Normalized times devices for the top device in the fourth leg for all the four cases of t_d.

Sector	I	II	III	IV	V	VI
$Z_{dp\,(p)}$	$t_{10} + t_8$	$t_{10} + t_8$	$t_{10} + t_8$	$t_{10} + t_8$	$t_{10} + t_8$	$t_{10} + t_8$
$Z_{dp\,(n)}$	$t_a + t_b + t_{10} + t_8$	$t_a + t_b + t_{10} + t_8$	$t_a + t_b + t_{10} + t_8$	$t_a + t_b + t_{10} + t_8$	$t_a + t_b + t_{10} + t_8$	$t_a + t_b + t_{10} + t_8$
Z_{dp} (CaseI)	$t_b + t_{10} + t_8$	$t_b + t_{10} + t_8$	$t_b + t_{10} + t_8$	$t_b + t_{10} + t_8$	$t_b + t_{10} + t_8$	$t_b + t_{10} + t_8$
Z_{dp} (CaseII)	$t_a + t_{10} + t_8$	$t_a + t_{10} + t_8$	$t_a + t_{10} + t_8$	$t_a + t_{10} + t_8$	$t_a + t_{10} + t_8$	$t_a + t_{10} + t_8$

Substituting the equation (5.10) and those in Table 5.3 and 5.5 into Table 5.6 the equations for modulating signals in all the sectors can be laid out as follows:

Consider Sector I:

$$Z_{ap} = t_a + t_b + t_9 + t_8 = \kappa t_c + \gamma t_d + \frac{V_{ac}}{V_d} + \frac{V_{cb}}{V_d}$$

Now $t_c = 1 - (t_a + t_b + t_d)$ and $t_a + t_b = \frac{V_a - V_c}{V_d} + \frac{V_c - V_b}{V_d} = \frac{V_{ab}}{V_d} = V_{abn}$

Thus re arranging the terms we have $Z_{ap} = V_{abn} + \gamma t_d + \kappa(1 - V_{abn} - t_d)$

$$Z_{ap} = t_9 + t_8 = \kappa t_c + \gamma t_d = \kappa(1 - t_a - t_d - t_d) + \gamma t_d = \gamma t_d + \kappa(1 - V_{abn} - t_d)$$

$$Z_{cp} = t_b + t_9 + t_8 = \kappa(1 - V_{abn} - t_d) + V_{cbn} + \gamma t_d$$

Now

$$Z_{ap} = t_{10} + t_8 = \gamma t_d + \kappa(1 - V_{abn} - t_d) + (1-\gamma)t_d$$

$$Z_{dp(n)} = t_a + t_b + t_{10} + t_8 = V_{abn} + \kappa(1 - V_{abn} - t_d) + (1-\gamma)t_d$$

$$Z_{dp(caseI)} = t_b + t_{10} + t_8 = V_{cbn} + \kappa(1 - V_{abn} - t_d) + (1-\gamma)t_d$$

$$Z_{dp(caseII)} = t_a + t_{10} + t_8 = V_{acn} + \kappa(1 - V_{abn} - t_d) + (1-\gamma)t_d$$

The same approach is used to derive the expressions for the remaining sectors

$t_a + t_b + t_c + t_d = 1 \qquad t_c = 1 - t_a - t_b - t_d$

where $t_c = t_7 + t_8$ and

$t_d = t_9 + t_{10}$

let $t_8 = kt_c$

$\therefore t_7 = (1-k)t_c$

and $t_9 = \gamma t_d$

$t_{10} = (1-\gamma)t_d$

Consider sector II

$Z_{ap} = t_8 + t_9 + t_a$

from Table 3.14 $\quad t_a = \dfrac{V_{ab}}{V_d} \quad t_b = \dfrac{V_{ca}}{V_d}$

$$= kt_c + \gamma t_d + \frac{V_{ab}}{V_d}$$

$$= k\left(1 - \frac{V_{ab}}{V_d} - \frac{V_{ca}}{V_d} - t_d\right) + \gamma t_d + \frac{V_{ab}}{V_d}$$

$$= k\left(1 - \frac{V_{cb}}{V_d} - t_d\right) + \gamma t_d + \frac{V_{ab}}{V_d}$$

$$Z_{ap} = k(1 - V_{cbn} - t_d) + \gamma t_d + V_{abn}$$

$Z_{bp} = t_8 + t_9$

$= kt_c + \gamma t_d$

$= k(1 - t_a - t_b - t_d) + \gamma t_d$

$$= k(1 - \frac{V_{ab}}{V_d} - \frac{V_{ca}}{V_d} - t_d) + \gamma t_d$$

$$Z_{bp} = k(1 - V_{cbn} - t_d) + \gamma t_d$$

$$Z_{cp} = t_8 + t_{10} + t_a + t_b$$

$$= kt_c + (1-\gamma)t_d + \frac{V_{ab}}{V_d} + \frac{V_{ca}}{V_d}$$

$$= k(1 - V_{cbn} - t_d) + (1-\gamma)t_d + \frac{V_{cb}}{V_d}$$

$$Z_{cp} = k(1 - V_{cbn} - t_d) + (1-\gamma)t_d + V_{cbn}$$

Positive sequence:

$$Z_{dp} = t_8 + t_{10}$$

$$= kt_c + (1-\gamma)t_d$$

$$Z_{dp} = k(1 - V_{cbn} - t_d) + (1-\gamma)t_d$$

Negative Sequence:

$$Z_{dp} = t_8 + t_{10} + t_a + t_b$$

$$= kt_c + (1-\gamma)t_d + t_a + t_b$$

$$= k(1 - V_{cbn} - t_d) + (1-\gamma)t_d + \frac{V_{ab}}{V_d} + \frac{V_{ca}}{V_d}$$

$$Z_{dp} = k(1 - V_{cbn} - t_d) + (1-\gamma)t_d + V_{cbn}$$

Case I

$$Z_{dp} = t_8 + t_{10} + t_b$$

$$= kt_c + (1-\gamma)t_d + t_b$$

$$= k(1 - V_{abn} - t_d) + (1-\gamma)t_d + \frac{V_{ca}}{V_d}$$

$$Z_{dp} = k(1 - V_{abn} - t_d) + (1 - \gamma)t_d + V_{can}$$

Case II

$$Z_{dp} = t_8 + t_9 + t_a$$

$$= kt_c + \gamma t_d + t_a$$

$$= k(1 - V_{abn} - t_d) + \gamma t_d + \frac{V_{ab}}{V_d}$$

$$Z_{dp} = k(1 - V_{abn} - t_d) + (1 - \gamma)t_d + V_{abn}$$

Consider sector III

$$Z_{ap} = t_8 + t_9$$

from Table 3.14 $\quad t_a = \dfrac{V_{cb}}{V_d} \quad t_b = \dfrac{V_{ba}}{V_d}$

$$= kt_c + \gamma t_d$$

$$= k(1 - \frac{V_{cb}}{V_d} - \frac{V_{ba}}{V_d} - t_d) + \gamma t_d$$

$$= k(1 - \frac{V_{ca}}{V_d} - t_d) + \gamma t_d$$

$$Z_{ap} = k(1 - V_{can} - t_d) + \gamma t_d$$

$$Z_{bp} = t_8 + t_9 + t_b$$

$$= kt_c + \gamma t_d + \frac{V_{ba}}{V_d}$$

$$= k(1 - t_a - t_b - t_d) + \gamma t_d + V_{ban}$$

$$= k(1 - \frac{V_{cb}}{V_d} - \frac{V_{ba}}{V_d} - t_d) + \gamma t_d + V_{ban}$$

$$Z_{bp} = k(1 - V_{can} - t_d) + \gamma t_d + V_{ban}$$

$$Z_{cp} = t_8 + t_9 + t_a + t_b$$

$$= kt_c + \gamma t_d + \frac{V_{cb}}{V_d} + \frac{V_{ba}}{V_d}$$

$$= k(1 - V_{can} - t_d) + \gamma t_d + \frac{V_{ca}}{V_d}$$

$$Z_{cp} = k(1 - V_{can} - t_d) + \gamma t_d + V_{can}$$

Positive sequence:

$$Z_{dp} = t_8 + t_{10}$$

$$= kt_c + (1 - \gamma)t_d$$

$$Z_{dp} = k(1 - V_{can} - t_d) + (1 - \gamma)t_d$$

Negative Sequence:

$$Z_{dp} = t_8 + t_{10} + t_a + t_b$$

$$= kt_c + (1 - \gamma)t_d + t_a + t_b$$

$$= k(1 - V_{can} - t_d) + (1 - \gamma)t_d + \frac{V_{cb}}{V_d} + \frac{V_{ba}}{V_d}$$

$$Z_{dp} = k(1 - V_{can} - t_d) + (1 - \gamma)t_d + V_{can}$$

Case I

$$Z_{dp} = t_8 + t_{10} + t_b$$

$$= kt_c + (1 - \gamma)t_d + t_b$$

$$= k(1 - V_{can} - t_d) + (1 - \gamma)t_d + \frac{V_{ba}}{V_d}$$

$$Z_{dp} = k(1 - V_{can} - t_d) + (1 - \gamma)t_d + V_{ban}$$

Case II

$$Z_{dp} = t_8 + t_9 + t_a$$

$$= kt_c + \gamma t_d + t_a$$

$$= k(1 - V_{cbn} - t_d) + \gamma t_d + \frac{V_{cb}}{V_d}$$

$$Z_{dp} = k(1 - V_{cbn} - t_d) + (1-\gamma)t_d + V_{cbn}$$

Consider sector IV

$$Z_{ap} = t_8 + t_9$$

from Table 3.14 $t_a = \frac{V_{ca}}{V_d}$ $t_b = \frac{V_{bc}}{V_d}$

$$= kt_c + \gamma t_d$$

$$= k(1 - \frac{V_{ca}}{V_d} - \frac{V_{bc}}{V_d} - t_d) + \gamma t_d$$

$$= k(1 - \frac{V_{ba}}{V_d} - t_d) + \gamma t_d$$

$$Z_{ap} = k(1 - V_{ban} - t_d) + \gamma t_d$$

$$Z_{bp} = t_8 + t_9 + t_b + t_a$$

$$= kt_c + \gamma t_d + \frac{V_{ca}}{V_d} + \frac{V_{bc}}{V_d}$$

$$= k(1 - t_a - t_b - t_d) + \gamma t_d + V_{ban}$$

$$= k(1 - \frac{V_{ca}}{V_d} - \frac{V_{bc}}{V_d} - t_d) + \gamma t_d + V_{ban}$$

$$Z_{bp} = k(1 - V_{ban} - t_d) + \gamma t_d + V_{ban}$$

$$Z_{cp} = t_8 + t_9 + t_a$$

$$= kt_c + \gamma t_d + \frac{V_{ca}}{V_d}$$

$$= k(1 - V_{ban} - t_d) + \gamma t_d + \frac{V_{ca}}{V_d}$$

$$Z_{cp} = k(1 - V_{ban} - t_d) + \gamma t_d + V_{can}$$

Positive sequence:

$$Z_{dp} = t_8 + t_{10}$$

$$= kt_c + (1-\gamma)t_d$$

$$Z_{dp} = k(1 - V_{ban} - t_d) + (1-\gamma)t_d$$

Negative Sequence:

$$Z_{dp} = t_8 + t_{10} + t_a + t_b$$

$$= kt_c + (1-\gamma)t_d + t_a + t_b$$

$$= k(1 - V_{ban} - t_d) + (1-\gamma)t_d + \frac{V_{ca}}{V_d} + \frac{V_{bc}}{V_d}$$

$$Z_{dp} = k(1 - V_{ban} - t_d) + (1-\gamma)t_d + V_{ban}$$

Case I

$$Z_{dp} = t_8 + t_{10} + t_b$$

$$= kt_c + (1-\gamma)t_d + t_b$$

$$= k(1 - V_{ban} - t_d) + (1-\gamma)t_d + \frac{V_{bc}}{V_d}$$

$$Z_{dp} = k(1 - V_{ban} - t_d) + (1-\gamma)t_d + V_{bcn}$$

Case II

$$Z_{dp} = t_8 + t_9 + t_a$$

$$= kt_c + \gamma t_d + t_a$$

$$= k(1 - V_{ban} - t_d) + \gamma t_d + \frac{V_{ca}}{V_d}$$

$$Z_{dp} = k(1 - V_{cbn} - t_d) + (1-\gamma)t_d + V_{can}$$

Consider sector V

$$Z_{ap} = t_8 + t_9 + t_b$$

from Table 3.14 $\quad t_a = \dfrac{V_{ba}}{V_d} \quad t_b = \dfrac{V_{ac}}{V_d}$

$$= kt_c + \gamma t_d + t_b$$

$$= k(1 - \frac{V_{ba}}{V_d} - \frac{V_{ac}}{V_d} - t_d) + \gamma t_d + \frac{V_{ac}}{V_d}$$

$$= k(1 - \frac{V_{bc}}{V_d} - t_d) + \gamma t_d + V_{acn}$$

$$Z_{ap} = k(1 - V_{bcn} - t_d) + \gamma t_d + V_{acn}$$

$$Z_{bp} = t_8 + t_9 + t_b + t_a$$

$$= kt_c + \gamma t_d + \frac{V_{ba}}{V_d} + \frac{V_{ac}}{V_d}$$

$$= k(1 - t_a - t_b - t_d) + \gamma t_d + V_{acn}$$

$$= k(1 - \frac{V_{ba}}{V_d} - \frac{V_{ac}}{V_d} - t_d) + \gamma t_d + V_{acn}$$

$$Z_{bp} = k(1 - V_{bcn} - t_d) + \gamma t_d + V_{ban}$$

$$Z_{cp} = t_8 + t_9 + t_a$$

$$= kt_c + \gamma t_d + \frac{V_{ba}}{V_d}$$

$$= k(1 - V_{bcn} - t_d) + \gamma t_d + \frac{V_{ba}}{V_d}$$

$$Z_{cp} = k(1 - V_{bcn} - t_d) + \gamma t_d + V_{ban}$$

Positive sequence:

$$Z_{dp} = t_8 + t_{10}$$

$$= kt_c + (1 - \gamma)t_d$$

$$Z_{dp} = k(1 - V_{bcn} - t_d) + (1 - \gamma)t_d$$

Negative Sequence:

$$Z_{dp} = t_8 + t_{10} + t_a + t_b$$

$$= kt_c + (1-\gamma)t_d + t_a + t_b$$

$$= k(1 - V_{bcn} - t_d) + (1-\gamma)t_d + \frac{V_{ba}}{V_d} + \frac{V_{ac}}{V_d}$$

$$Z_{dp} = k(1 - V_{bcn} - t_d) + (1-\gamma)t_d + V_{acn}$$

Case I

$$Z_{dp} = t_8 + t_{10} + t_b$$

$$= kt_c + (1-\gamma)t_d + t_b$$

$$= k(1 - V_{bcn} - t_d) + (1-\gamma)t_d + \frac{V_{ac}}{V_d}$$

$$Z_{dp} = k(1 - V_{bcn} - t_d) + (1-\gamma)t_d + V_{acn}$$

Case II

$$Z_{dp} = t_8 + t_9 + t_a$$

$$= kt_c + \gamma t_d + t_a$$

$$= k(1 - V_{bcn} - t_d) + \gamma t_d + \frac{V_{ba}}{V_d}$$

$$Z_{dp} = k(1 - V_{bcn} - t_d) + (1-\gamma)t_d + V_{ban}$$

Consider Sector VI

$$Z_{ap} = t_8 + t_9 + t_b + t_a$$

from Table 3.14 $\quad t_a = \frac{V_{bc}}{V_d} \;\; t_b = \frac{V_{ab}}{V_d}$

$$= kt_c + \gamma t_d + t_b + t_a$$

$$= k(1 - \frac{V_{bc}}{V_d} - \frac{V_{ab}}{V_d} - t_d) + \gamma t_d + \frac{V_{bc}}{V_d} + \frac{V_{ab}}{V_d}$$

$$= k(1 - \frac{V_{ac}}{V_d} - t_d) + \gamma t_d + V_{acn}$$

$$Z_{ap} = k(1 - V_{acn} - t_d) + \gamma t_d + V_{acn}$$

$$Z_{bp} = t_8 + t_9 + t_a$$

$$= kt_c + \gamma t_d + \frac{V_{bc}}{V_d}$$

$$= k(1 - t_a - t_b - t_d) + \gamma t_d + V_{bcn}$$

$$= k(1 - \frac{V_{bc}}{V_d} - \frac{V_{ab}}{V_d} - t_d) + \gamma t_d + V_{bcn}$$

$$Z_{bp} = k(1 - V_{acn} - t_d) + \gamma t_d + V_{bcn}$$

$$Z_{cp} = t_8 + t_9$$

$$= kt_c + \gamma t_d$$

$$= k(1 - V_{acn} - t_d) + \gamma t_d$$

$$Z_{cp} = k(1 - V_{acn} - t_d) + \gamma t_d$$

Positive sequence:

$$Z_{dp} = t_8 + t_{10}$$

$$= kt_c + (1-\gamma)t_d$$

$$Z_{dp} = k(1 - V_{acn} - t_d) + (1-\gamma)t_d$$

Negative Sequence:

$$Z_{dp} = t_8 + t_{10} + t_a + t_b$$

$$= kt_c + (1-\gamma)t_d + t_a + t_b$$

$$= k(1 - V_{acn} - t_d) + (1-\gamma)t_d + \frac{V_{bc}}{V_d} + \frac{V_{ab}}{V_d}$$

$$Z_{dp} = k(1 - V_{acn} - t_d) + (1-\gamma)t_d + V_{acn}$$

Case I

$$Z_{dp} = t_8 + t_{10} + t_b$$

$$= kt_c + (1-\gamma)t_d + t_b$$

$$= k(1 - V_{acn} - t_d) + (1-\gamma)t_d + \frac{V_{ab}}{V_d}$$

$$Z_{dp} = k(1 - V_{acn} - t_d) + (1-\gamma)t_d + V_{abn}$$

Case II

$$Z_{dp} = t_8 + t_9 + t_a$$

$$= kt_c + \gamma t_d + t_a$$

$$= k(1 - V_{acn} - t_d) + \gamma t_d + \frac{V_{bc}}{V_d}$$

$$Z_{dp} = k(1 - V_{acn} - t_d) + (1-\gamma)t_d + V_{bcn}$$

Summary of the generalized expression derived is shown in Table 5.7

Table 5.7 Modulation signals for the top devices [5.12].

	Map	Mbp	Mcp	Mdp(p)	Mdp(n)	Mdp(I)	Md(II)
	$V_{abn}+\gamma t_d+$ $\kappa(1-V_{abn}-t_d)$	γt_d+ $\kappa(1-V_{abn}-t_d)$	$V_{abn}+\gamma t_d+$ $\kappa(1-V_{abn}-t_d)$	$V_{abn}+\gamma t_d+$ $\kappa(1-V_{abn}-t_d)$	$V_{abn}+\gamma t_d+$ $\kappa(1-V_{abn}-t_d)$	$V_{cbn}+\gamma t_d+$ $\kappa(1-V_{abn}-t_d)$	$V_{acn}+\gamma t_d+$ $\kappa(1-V_{abn}-t_d)$
1(	$V_{abn}+\gamma t_d+$ $\kappa(1-V_{cbn}-t_d)$	$\gamma t_d+\kappa(V_{cbn}-t_d)$	$V_{cbn}+\gamma t_d+$ $\kappa(1-V_{abn}-t_d)$	$V_{cbn}+\kappa(1-V_{cbn}-t_d)$ $+(1-\gamma)t_d$	$V_{cbn}+\kappa(1-V_{cbn}-t_d)$ $+(1-\gamma)t_d$	$V_{cbn}+\kappa(1-V_{cbn}-t_d)$ $+(1-\gamma)t_d$	$V_{abn}+\kappa(1-V_{cbn}-$ $+(1-\gamma)t_d$
0	$\gamma t_d+\kappa(1-V_{can}-t_d)$	$V_{ban}+\gamma t_d+$ $\kappa(1-V_{can}-t_d)$	$V_{can}+\gamma t_d+$ $\kappa(1-V_{can}-t_d)$	$V_{cbn}+\kappa(1-V_{ban}-t_d)$ $+(1-\gamma)t_d$	$V_{can}+\kappa(1-V_{can}-t_d)$ $+(1-\gamma)t_d$	$V_{bsn}+\kappa(1-V_{van}-t_d)$ $+(1-\gamma)t_d$	$V_{cbn}+\kappa(1-V_{ca}$ $+(1-\gamma)t_d$
20	$\gamma t_d+\kappa(V_{ban}-t_d)$	$V_{ban}+\gamma t_d+$ $\kappa(1-V_{can}-t_d)$	$V_{can}+\gamma t_d+$ $\kappa(1-V_{ban}-t_d)$	$V_{ban}+\kappa(1-V_{abn}-t_d)$ $+(1-\gamma)t_d$	$V_{ban}+\kappa(1-V_{ban}-t_d)$ $+(1-\gamma)t_d$	$V_{bcn}+\kappa(1-V_{ban}-t_d)$ $+(1-\gamma)t_d$	$V_{can}+\kappa(1-V_{ban}$ $+(1-\gamma)t_d$
60	$V_{cbn}+\gamma t_d+$ $\kappa(1-V_{bcn}-t_d)$	$V_{can}+\gamma t_d+$ $\kappa(1-V_{ban}-t_d)$	γt_d+ $\kappa(1-V_{bcn}-t_d)$	$V_{bcn}+\kappa(1-V_{bcn}-t_d)$ $+(1-\gamma)t_d$	$V_{bcn}+\kappa(1-V_{bcn}-t_d)$ $+(1-\gamma)t_d$	$V_{acn}+\kappa(1-V_{bcn}-t_d)$ $+(1-\gamma)t_d$	$V_{ban}+\kappa(1-V_{b}$ $+(1-\gamma)t_d$
0	$V_{bcn}+\gamma t_d+$ $\kappa(1-V_{acn}-t_d)$	γt_d+ $\kappa(1-V_{bcn}-t_d)$	γt_d+ $\kappa(1-V_{acn}-t_d)$	$V_{acn}+\kappa(1-V_{acn}-t_d)$ $+(1-\gamma)t_d$	$V_{acn}+\kappa(1-V_{acn}-t_d)$ $+(1-\gamma)t_d$	$V_{abn}+\kappa(1-V_{acn}-t_d)$ $+(1-\gamma)t_d$	$V_{bcn}+\kappa(1-V_{ac}$ $+(1-\gamma)t_d$

Table 5.8 (a) Switching combinations for the top devices in positive sequence

Positive Sequence

k = 1, r = 1

SECTOR I				
	Sa	Sb	Sc	Sd
ta	1	0	0	0
tb	1	0	1	0
t9	1	1	1	0
t8	1	1	1	1

k = 0, r = 1

SECTOR I				
	Sa	Sb	Sc	Sd
ta	1	0	0	0
tb	1	0	1	0
t9	1	1	1	0
t7	0	0	0	0

k = 0, r = 0

SECTOR I				
	Sa	Sb	Sc	Sd
ta	1	0	0	0
tb	1	0	1	0
t10	0	0	0	1
t7	0	0	0	0

k = 1, r = 0

SECTOR I				
	Sa	Sb	Sc	Sd
ta	1	0	0	0
tb	1	0	1	0
t10	0	0	0	1
t8	1	1	1	1

k = 1, r = 1

SECTOR II				
	Sa	Sb	Sc	Sd
ta	1	0	1	0
tb	0	0	1	0
t9	1	1	1	0
t8	1	1	1	1

k = 0, r = 1

SECTOR II				
	Sa	Sb	Sc	Sd
ta	1	0	1	0
tb	0	0	1	0
t9	1	1	1	0
t7	0	0	0	0

k = 0, r = 0

SECTOR II				
	Sa	Sb	Sc	Sd
ta	1	0	1	0
tb	0	0	1	0
t10	0	0	0	1
t7	0	0	0	0

k = 1, r = 0

SECTOR II				
	Sa	Sb	Sc	Sd
ta	1	0	1	0
tb	0	0	1	0
t10	0	0	0	1
t8	1	1	1	1

k = 1, r = 1

SECTOR III				
	Sa	Sb	Sc	Sd
ta	0	0	1	0
tb	0	1	1	0
t9	1	1	1	0
t8	1	1	1	1

k = 0, r = 1

SECTOR III				
	Sa	Sb	Sc	Sd
ta	0	0	1	0
tb	0	1	1	0
t9	1	1	1	0
t7	0	0	0	0

k = 0, r = 0

SECTOR III				
	Sa	Sb	Sc	Sd
ta	0	0	1	0
tb	0	1	1	0
t10	0	0	0	1
t7	0	0	0	0

k = 1, r = 0

SECTOR III				
	Sa	Sb	Sc	Sd
ta	0	0	1	0
tb	0	1	1	0
t10	0	0	0	1
t8	1	1	1	1

k = 1, r = 1

SECTOR IV				
	Sa	Sb	Sc	Sd
ta	0	1	1	0
tb	0	1	0	0
t9	1	1	1	0
t8	1	1	1	1

k = 0, r = 1

SECTOR IV				
	Sa	Sb	Sc	Sd
ta	0	1	1	0
tb	0	1	0	0
t9	1	1	1	0
t7	0	0	0	0

k = 0, r = 0

SECTOR IV				
	Sa	Sb	Sc	Sd
ta	0	1	1	0
tb	0	1	0	0
t10	0	0	0	1
t7	0	0	0	0

k = 1, r = 0

SECTOR IV				
	Sa	Sb	Sc	Sd
ta	0	1	1	0
tb	0	1	0	0
t10	0	0	0	1
t8	1	1	1	1

k = 1, r = 1

SECTOR V				
	Sa	Sb	Sc	Sd
ta	0	1	0	0
tb	1	1	0	0
t9	1	1	1	0
t8	1	1	1	1

k = 0, r = 1

SECTOR V				
	Sa	Sb	Sc	Sd
ta	0	1	0	0
tb	1	1	0	0
t9	1	1	1	0
t7	0	0	0	0

k = 0, r = 0

SECTOR V				
	Sa	Sb	Sc	Sd
ta	0	1	0	0
tb	1	1	0	0
t10	0	0	0	1
t7	0	0	0	0

k = 1, r = 0

SECTOR V				
	Sa	Sb	Sc	Sd
ta	0	1	0	0
tb	1	1	0	0
t10	0	0	0	1
t8	1	1	1	1

k = 1, r = 1

SECTOR VI				
	Sa	Sb	Sc	Sd
ta	1	1	0	0
tb	1	0	0	0
t9	1	1	1	0
t8	1	1	1	1

k = 0, r = 1

SECTOR VI				
	Sa	Sb	Sc	Sd
ta	1	1	0	0
tb	1	0	0	0
t9	1	1	1	0
t7	0	0	0	0

k = 0, r = 0

SECTOR VI				
	Sa	Sb	Sc	Sd
ta	1	1	0	0
tb	1	0	0	0
t10	0	0	0	1
t7	0	0	0	0

k = 1, r = 0

SECTOR VI				
	Sa	Sb	Sc	Sd
ta	1	1	0	0
tb	1	0	0	0
t10	0	0	0	1
t8	1	1	1	1

Table 5.8 (b) Switching combinations for the top devices in negative sequence

Negative Sequence

k=0, r=0

SECTOR I				
	Sa	Sb	Sc	Sd
ta	1	0	0	1
tb	1	0	1	1
t10	0	0	0	1
t7	0	0	0	0

k=1, r=0

SECTOR I				
	Sa	Sb	Sc	Sd
ta	1	0	0	1
tb	1	0	1	1
t10	0	0	0	1
t8	1	1	1	1

k=1, r=1

SECTOR I				
	Sa	Sb	Sc	Sd
ta	1	0	0	1
tb	1	0	1	1
t9	1	1	1	0
t8	1	1	1	1

k=0, r=1

SECTOR I				
	Sa	Sb	Sc	Sd
ta	1	0	0	1
tb	1	0	1	1
t9	1	1	1	0
t7	0	0	0	0

k=0, r=0

SECTOR II				
	Sa	Sb	Sc	Sd
ta	1	0	1	1
tb	0	0	1	1
t10	0	0	0	1
t7	0	0	0	0

k=1, r=0

SECTOR II				
	Sa	Sb	Sc	Sd
ta	1	0	1	1
tb	0	0	1	1
t10	0	0	0	1
t8	1	1	1	1

k=1, r=1

SECTOR II				
	Sa	Sb	Sc	Sd
ta	1	0	1	1
tb	0	0	1	1
t9	1	1	1	0
t8	1	1	1	1

k=0, r=1

SECTOR II				
	Sa	Sb	Sc	Sd
ta	1	0	1	1
tb	0	0	1	1
t9	1	1	1	0
t7	0	0	0	0

k=0, r=0

SECTOR III				
	Sa	Sb	Sc	Sd
ta	0	0	1	1
tb	0	1	1	1
t10	0	0	0	1
t7	0	0	0	0

k=1, r=0

SECTOR III				
	Sa	Sb	Sc	Sd
ta	0	0	1	1
tb	0	1	1	1
t10	0	0	0	1
t8	1	1	1	1

k=1, r=1

SECTOR III				
	Sa	Sb	Sc	Sd
ta	0	0	1	1
tb	0	1	1	1
t9	1	1	1	0
t8	1	1	1	1

k=0, r=1

SECTOR III				
	Sa	Sb	Sc	Sd
ta	0	0	1	1
tb	0	1	1	1
t9	1	1	1	0
t7	0	0	0	0

k=0, r=0

SECTOR IV				
	Sa	Sb	Sc	Sd
ta	0	1	1	1
tb	0	1	0	1
t10	0	0	0	1
t7	0	0	0	0

k=1, r=0

SECTOR IV				
	Sa	Sb	Sc	Sd
ta	0	1	1	1
tb	0	1	0	1
t10	0	0	0	1
t8	1	1	1	1

k=1, r=1

SECTOR IV				
	Sa	Sb	Sc	Sd
ta	0	1	1	1
tb	0	1	0	1
t9	1	1	1	0
t8	1	1	1	1

k=0, r=1

SECTOR IV				
	Sa	Sb	Sc	Sd
ta	0	1	1	1
tb	0	1	0	1
t9	1	1	1	0
t7	0	0	0	0

k=0, r=0

SECTOR V				
	Sa	Sb	Sc	Sd
ta	0	1	0	1
tb	1	1	0	1
t10	0	0	0	1
t7	0	0	0	0

k=1, r=0

SECTOR V				
	Sa	Sb	Sc	Sd
ta	0	1	0	1
tb	1	1	0	1
t10	0	0	0	1
t8	1	1	1	1

k=1, r=1

SECTOR V				
	Sa	Sb	Sc	Sd
ta	0	1	0	1
tb	1	1	0	1
t9	1	1	1	0
t8	1	1	1	1

k=0, r=1

SECTOR V				
	Sa	Sb	Sc	Sd
ta	0	1	0	1
tb	1	1	0	1
t9	1	1	1	0
t7	0	0	0	0

k=0, r=0

SECTOR VI				
	Sa	Sb	Sc	Sd
ta	1	1	0	1
tb	1	0	0	1
t10	0	0	0	1
t7	0	0	0	0

k=1, r=0

SECTOR VI				
	Sa	Sb	Sc	Sd
ta	1	1	0	1
tb	1	0	0	1
t10	0	0	0	1
t8	1	1	1	1

k=1, r=1

SECTOR VI				
	Sa	Sb	Sc	Sd
ta	1	1	0	1
tb	1	0	0	1
t9	1	1	1	0
t8	1	1	1	1

k=0, r=1

SECTOR VI				
	Sa	Sb	Sc	Sd
ta	1	1	0	1
tb	1	0	0	1
t9	1	1	1	0
t7	0	0	0	0

Table 5.8 (c) Switching combinations for the top devices CaseI

k =1, r = 1

SECTOR I				
	Sa	Sb	Sc	Sd
ta	1	0	0	0
tb	1	0	1	1
t9	1	1	1	0
t8	1	1	1	1

k = 0, r = 1

SECTOR I				
	Sa	Sb	Sc	Sd
ta	1	0	0	0
tb	1	0	1	1
t9	1	1	1	0
t7	0	0	0	0

k = 0, r = 0

SECTOR I				
	Sa	Sb	Sc	Sd
ta	1	0	0	0
tb	1	0	1	1
t10	0	0	0	1
t7	0	0	0	0

k = 1, r = 0

SECTOR I				
	Sa	Sb	Sc	Sd
ta	1	0	0	0
tb	1	0	1	1
t10	0	0	0	1
t8	1	1	1	1

k =1, r = 1

SECTOR II				
	Sa	Sb	Sc	Sd
ta	1	0	1	0
tb	0	0	1	1
t9	1	1	1	0
t8	1	1	1	1

k = 0, r =1

SECTOR II				
	Sa	Sb	Sc	Sd
ta	1	0	1	0
tb	0	0	1	1
t9	1	1	1	0
t7	0	0	0	0

k = 0, r = 0

SECTOR II				
	Sa	Sb	Sc	Sd
ta	1	0	1	0
tb	0	0	1	1
t10	0	0	0	1
t7	0	0	0	0

k = 1, r = 0

SECTOR II				
	Sa	Sb	Sc	Sd
ta	1	0	1	0
tb	0	0	1	1
t10	0	0	0	1
t8	1	1	1	1

k =1, r = 1

SECTOR III				
	Sa	Sb	Sc	Sd
ta	0	0	1	0
tb	0	1	1	1
t9	1	1	1	0
t8	1	1	1	1

k = 0, r =1

SECTOR III				
	Sa	Sb	Sc	Sd
ta	0	0	1	0
tb	0	1	1	1
t9	1	1	1	0
t7	0	0	0	0

k = 0, r = 0

SECTOR III				
	Sa	Sb	Sc	Sd
ta	0	0	1	0
tb	0	1	1	1
t10	0	0	0	1
t7	0	0	0	0

k = 1, r = 0

SECTOR III				
	Sa	Sb	Sc	Sd
ta	0	0	1	0
tb	0	1	1	1
t10	0	0	0	1
t8	1	1	1	1

k =1, r = 1

SECTOR IV				
	Sa	Sb	Sc	Sd
ta	0	1	1	0
tb	0	1	0	1
t9	1	1	1	0
t8	1	1	1	1

k = 0, r =1

SECTOR IV				
	Sa	Sb	Sc	Sd
ta	0	1	1	0
tb	0	1	0	1
t9	1	1	1	0
t7	0	0	0	0

k = 0, r = 0

SECTOR IV				
	Sa	Sb	Sc	Sd
ta	0	1	1	0
tb	0	1	0	1
t10	0	0	0	1
t7	0	0	0	0

k = 1, r = 0

SECTOR IV				
	Sa	Sb	Sc	Sd
ta	0	1	1	0
tb	0	1	0	1
t10	0	0	0	1
t8	1	1	1	1

k =1, r = 1

SECTOR V				
	Sa	Sb	Sc	Sd
ta	0	1	0	0
tb	1	1	0	1
t9	1	1	1	0
t8	1	1	1	1

k = 0, r = 1

SECTOR V				
	Sa	Sb	Sc	Sd
ta	0	1	0	0
tb	1	1	0	1
t9	1	1	1	0
t7	0	0	0	0

k = 0, r = 0

SECTOR V				
	Sa	Sb	Sc	Sd
ta	0	1	0	0
tb	1	1	0	1
t10	0	0	0	1
t7	0	0	0	0

k = 1, r = 0

SECTOR V				
	Sa	Sb	Sc	Sd
ta	0	1	0	0
tb	1	1	0	1
t10	0	0	0	1
t8	1	1	1	1

k =1, r = 1

SECTOR VI				
	Sa	Sb	Sc	Sd
ta	1	1	0	0
tb	1	0	0	1
t9	1	1	1	0
t8	1	1	1	1

k = 0, r = 1

SECTOR VI				
	Sa	Sb	Sc	Sd
ta	1	1	0	0
tb	1	0	0	1
t9	1	1	1	0
t7	0	0	0	0

k = 0, r = 0

SECTOR VI				
	Sa	Sb	Sc	Sd
ta	1	1	0	0
tb	1	0	0	1
t10	0	0	0	1
t7	0	0	0	0

k = 1, r = 0

SECTOR VI				
	Sa	Sb	Sc	Sd
ta	1	1	0	0
tb	1	0	0	1
t10	0	0	0	1
t8	1	1	1	1

Table 5.8 (d) Switching combinations for the top devices CaseII

k =0, r =0

SECTOR I	Sa	Sb	Sc	Sd
ta	1	0	0	1
tb	1	0	1	0
t10	0	0	0	1
t7	0	0	0	0

k =1, r = 0

SECTOR I	Sa	Sb	Sc	Sd
ta	1	0	0	1
tb	1	0	1	0
t10	0	0	0	1
t8	1	1	1	1

k =1, r =1

SECTOR I	Sa	Sb	Sc	Sd
ta	1	0	0	1
tb	1	0	1	0
t9	1	1	1	0
t8	1	1	1	1

k =0, r =1

SECTOR I	Sa	Sb	Sc	Sd
ta	1	0	0	1
tb	1	0	1	0
t9	1	1	1	0
t7	0	0	0	0

k =0, r = 0

SECTOR II	Sa	Sb	Sc	Sd
ta	1	0	1	1
tb	0	0	1	0
t10	0	0	0	1
t7	0	0	0	0

k =1, r = 0

SECTOR II	Sa	Sb	Sc	Sd
ta	1	0	1	1
tb	0	0	1	0
t10	0	0	0	1
t8	1	1	1	1

k =1, r = 1

SECTOR II	Sa	Sb	Sc	Sd
ta	1	0	1	1
tb	0	0	1	0
t9	1	1	1	0
t8	1	1	1	1

k =0, r =1

SECTOR II	Sa	Sb	Sc	Sd
ta	1	0	1	1
tb	0	0	1	0
t9	1	1	1	0
t7	0	0	0	0

k =0, r = 0

SECTOR III	Sa	Sb	Sc	Sd
ta	0	0	1	1
tb	0	1	1	0
t10	0	0	0	1
t7	0	0	0	0

k =1, r = 0

SECTOR III	Sa	Sb	Sc	Sd
ta	0	0	1	1
tb	0	1	1	0
t10	0	0	0	1
t8	1	1	1	1

k =1, r = 1

SECTOR III	Sa	Sb	Sc	Sd
ta	0	0	1	1
tb	0	1	1	0
t9	1	1	1	0
t8	1	1	1	1

k =0, r =1

SECTOR III	Sa	Sb	Sc	Sd
ta	0	0	1	1
tb	0	1	1	0
t9	1	1	1	0
t7	0	0	0	0

k =0, r = 0

SECTOR IV	Sa	Sb	Sc	Sd
ta	0	1	1	1
tb	0	1	0	0
t10	0	0	0	1
t7	0	0	0	0

k =1, r = 0

SECTOR IV	Sa	Sb	Sc	Sd
ta	0	1	1	1
tb	0	1	0	0
t10	0	0	0	1
t8	1	1	1	1

k =1, r = 1

SECTOR IV	Sa	Sb	Sc	Sd
ta	0	1	1	1
tb	0	1	0	0
t9	1	1	1	0
t8	1	1	1	1

k =0, r =1

SECTOR IV	Sa	Sb	Sc	Sd
ta	0	1	1	1
tb	0	1	0	0
t9	1	1	1	0
t7	0	0	0	0

k =0, r = 0

SECTOR V	Sa	Sb	Sc	Sd
ta	0	1	0	1
tb	1	1	0	0
t10	0	0	0	1
t7	0	0	0	0

k =1, r = 0

SECTOR V	Sa	Sb	Sc	Sd
ta	0	1	0	1
tb	1	1	0	0
t10	0	0	0	1
t8	1	1	1	1

k =1, r = 1

SECTOR V	Sa	Sb	Sc	Sd
ta	0	1	0	1
tb	1	1	0	0
t9	1	1	1	0
t8	1	1	1	1

k =0, r =1

SECTOR V	Sa	Sb	Sc	Sd
ta	0	1	0	1
tb	1	1	0	0
t9	1	1	1	0
t7	0	0	0	0

k =0, r = 0

SECTOR VI	Sa	Sb	Sc	Sd
ta	1	1	0	1
tb	1	0	0	0
t10	0	0	0	1
t7	0	0	0	0

k =1, r = 0

SECTOR VI	Sa	Sb	Sc	Sd
ta	1	1	0	1
tb	1	0	0	0
t10	0	0	0	1
t8	1	1	1	1

k =1, r = 1

SECTOR VI	Sa	Sb	Sc	Sd
ta	1	1	0	1
tb	1	0	0	0
t9	1	1	1	0
t8	1	1	1	1

k =0, r =1

SECTOR VI	Sa	Sb	Sc	Sd
ta	1	1	0	1
tb	1	0	0	0
t9	1	1	1	0
t7	0	0	0	0

The equations for the modulation signals for the top devices are shown in Table 5.7. however, it is noted that the expressions for the modulation signal of the top devices in phases a,b,c are the same for the four switching mode combinations; but the expressions for the modulation signals for phase d and the normalized time t_d are different. In Tables 5.7 and 5.5, $M_{dp}(p)$ and $t_d(p)$, respectively, are the expressions for the top device d-phase modulation signal and time for the positive sequence space vector combination set while the corresponding expressions for the negative sequence space vector combination set are $M_{dp}(n)$ and $t_d(n)$, respectively. The corresponding expressions for the modulation signals and times of the other two combinations are $M_{dp}(I)$ and $t_d(I)$, $M_{dp}(II)$ and $t_d(II)$; respectively.

These normalized modulation signals for the devices are compared with a high frequency triangular carrier waveform ranging between unity and zero and the intersections define the device switching instants. A study of the switching combinations for all the four switching modes in each sector is shown in Tables 5.8 (a-d).

The active states t_a and t_b are combined with the available null states t_7 , t_8 , t_9 and t_{10}. $\gamma = 1$ means that the null state being used is t_9 while $\gamma = 0$ means that the null state used is t_{10}. Similarly $\kappa = 1$ corresponds to the null state t_8 while $\kappa = 0$ corresponds to null state t_7. Table 5.8 is a study of the switching sequence observed for various combinations of the null states with the active states.

These combinations in each sector reveals that the devices connected to phases a, b, c are clamped to the dc rail only when γ and κ take values of unity or zero and $\gamma = \kappa$. This is shown as highlighted region in tables 5.8 (a) through (d). In general, the value of γ [1,0] is selected to ensure that t_d and t_c are always positive. When $\kappa = 1- \gamma$, the d – phase device alone is clamped to the dc rail. Since switching devices connected to phases a, b, c carry most of the load currents; clamping these leads to the highest reduction of switching loss - condition $\gamma = \kappa$ appears to be the optimum selection.

It is noted that the selection of γ [1,0], κ[1,0] corresponding to the situation where only two null states are used in the synthesis – one with zero qdo value and another with zero qd but non-zero negative sequence voltage values is akin to the Class II sequencing 3-D SVM scheme set forth in [1]; which is shown to be the best compromise choice between switching losses and harmonic contents. An infinite number of possibilities results if γ, κ take fractional values – in which more than two of the members of the set of null states are utilized in the voltage synthesis – with loss of switching device clamping to either the positive of negative rails.

5.3.4 Synthesis of reference voltage

Given the instantaneous unbalanced three phase reference voltage set (equivalently, the qdo voltages in the stationary reference frame) the sector in Fig. 5.2 where it resides is determined. From Table 5.3, to assure positive value of t_d – positivity of time the value of γ is chosen, the times t_d for the four combinations are calculated and an appropriate switching combination that ensures that $t_c \geq 0$ is selected. There may be more than one combination that meets the time positivity requirement. Those possibilities of t_d which satisfy positivity requirement of t_c are stored as an array. Other constraints can be used to decide the selection of switching combination. This can be the minimum or maximum time t_d. Then Table 5.7 is used to determine the modulation signals for the top devices while those of the lower devices, as they are complementary to the top devices.

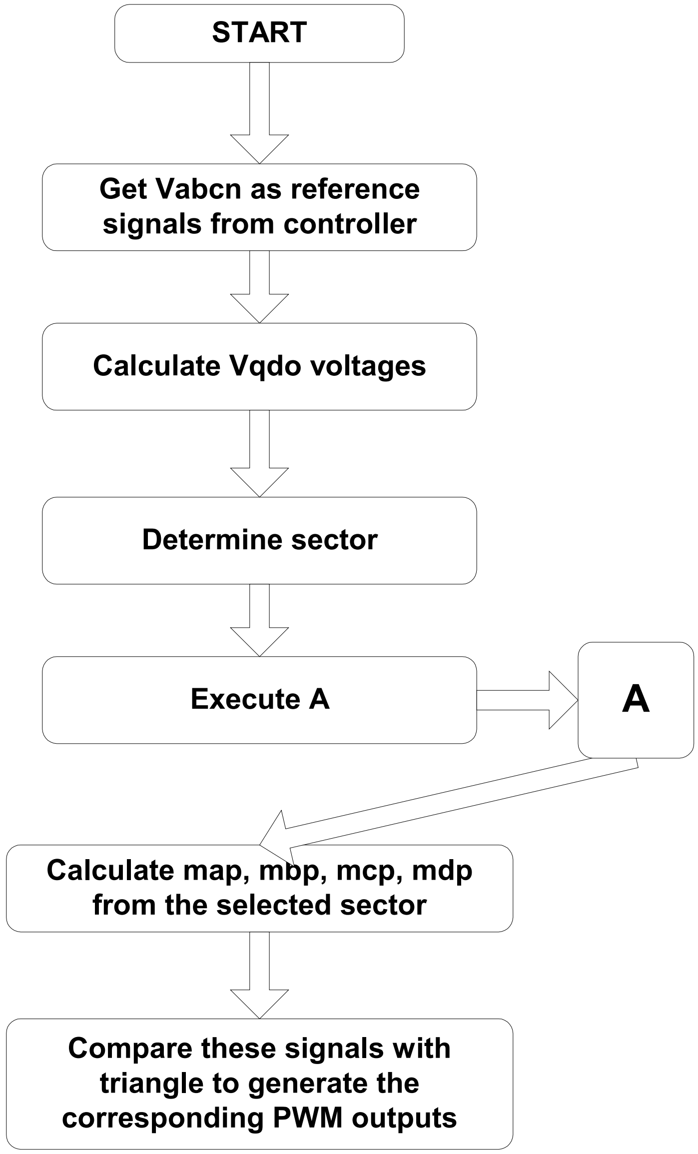


Figure 5.7 Synthesis of a reference voltage

5.5 Experimental results

The new carrier-based discontinuous PWM modulation scheme is implemented by means of a floating-point 40-MHz DSP TMS320LF2407 board to synthesize three-phase balanced and unbalanced phase voltages to a wye-connected three-phase load. While the maximum current and voltage of the four-leg converter used is 14A and 350V respectively, the DC voltage applied for the experiment is 60V and the phase of each of the three-phase load is comprised of a resistance of 40 Ohms shunted with a filter with an inductance of 14 mH and a capacitor of 30 μF.

The carrier based discontinuous scheme operation was studied with four different cases in synthesizing both balanced and unbalanced voltages

5.5.1 Synthesis of balanced reference voltages at modulation depth = 1 and 0.6

Figure 5.7 and 5.10 shows the step-by-step waveforms for algorithm shown in figure 5.6. Figure 5.7(a) and 5.10(a)

1. Scope 4: the stationary reference frame q-d voltages.
2. Scope 1: the a-b-c reference frame angle θ
3. Scope 3: the inverse tangent of the V_d/V_q.

Once the angle for stationary reference frame is determined the sectors are selected. This is shown in Figure 5.7 (b) and 5.10(b)

1. Scope 1: selection of sectors.
2. Scope 3: the inverse tangent of the V_d/V_q.

In every sector the absolute values of td is determined as shown in figure 5.7 (c) and 5.10(c)

1. Scope 1: Absolute of t_{dI}
2. Scope 2: Absolute of t_{dII}
3. Scope 3: Absolute of t_{dp}
4. Scope 41: Absolute of t_{dn}

In every sector the corresponding of t_c is calculated as shown in figure 5.7 (d) and 5.10(c)

1 Scope 1: Calculated $_{tcI}$

2 Scope 2: Calculated t_{cII}

3 Scope 3: Calculated t_{cp}

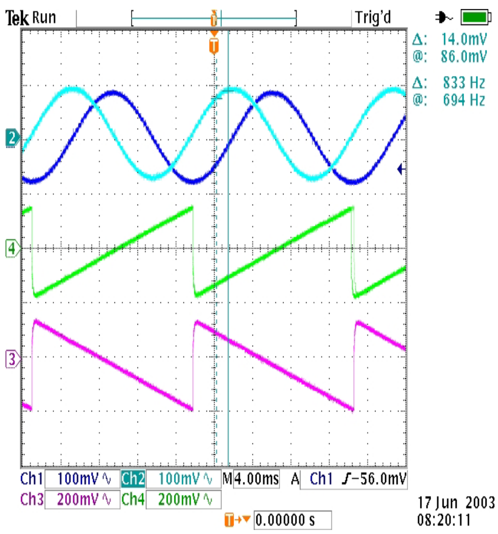
4 Scope 4: Calculated t_{cn}

After having the conditions of t_d and t_c it is seen in 5.7 (d) that t_{cp} and t_{cn} are zero. Hence we cannot use the corresponding td's i.e t_{dp} and t_{dn}. Thus the algorithm is designed to use t_{dI} and t_{dII} as shown in figure Figure 5.8(a) In 5.11(a) all the conditions of t_d and t_c are satisfied and hence the algorithm still permits the selection of maximum and minimum td for the given conditions of t_c.

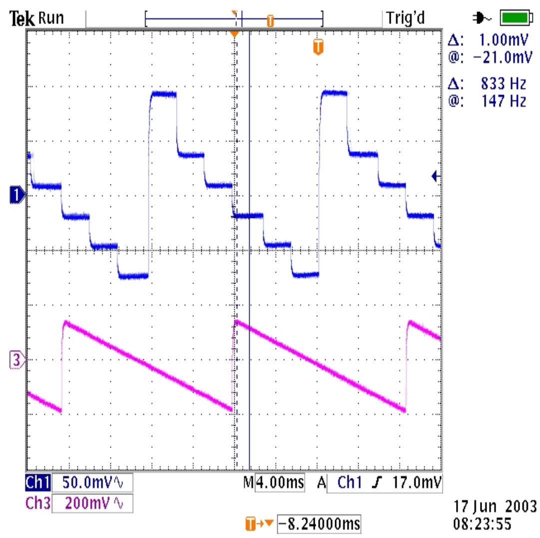
1. Scope 1: Condition for t_{dI}
2. Scope 2: Condition for t_{dII}
3. Scope 3: Selection of maximum t_{dmax}
4. Scope 4: Selection of minimum t_{dmin}

Out of the two conditions of td only one is used at a time to synthesize the reference voltage. The generated modulation signal for one of the top devices along with its switching is shown in Figures 5.8-5.11 (b) and (c) for the conditions of t_{dmax} and t_{dmin}. It can be seen from the switching that t_{dmin} produces less harmonics than t_{dmax}

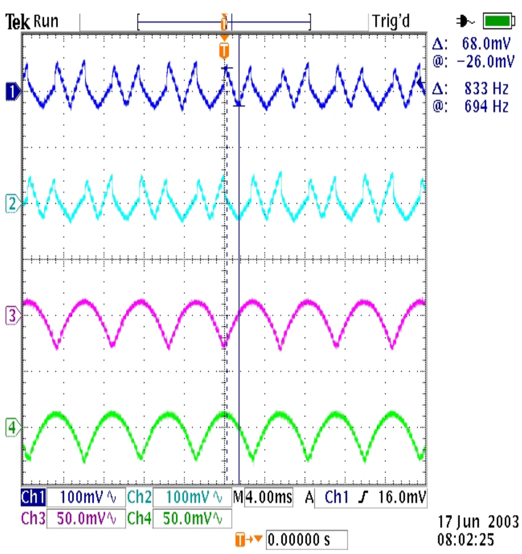
Figure 5.10(a) corresponds to the Class II(c) symmetrically aligned sequencing scheme of the proposed 3-D SVM in [5.7]. It is salutary to observe that 7(a) and 7(b) are respectively similar to the discontinuous modulation waveforms DPWM1 and DPWM3 of the three leg voltage source converters [6]. The generalized discontinuous PWM modulation (GDPWM) for the two level converters (DPWM1 and DPWM3 are members) by virtue of the injection of zero sequence signals to the sinusoidal PWM signals extends the PWM linearity range. So it is that the proposed modulation, especially for the switching mode combination satisfying the condition of minimum time t_d will improve the linearity range of the four-leg converter by increasing the voltage gain in the over-modulation region. For the case in which the three load voltages are balanced, the periods for which the devices in the four-leg converter are clamped in each load phase are the same (120 degrees cycle); however for the situation in which the load phase voltages are unbalanced, there is a total of 360 degree clamping for the three-phases which are unevenly distributed in the load phases.



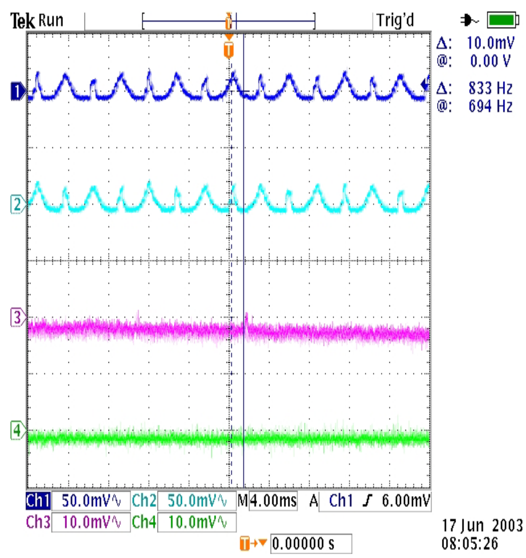
(a)



(b)

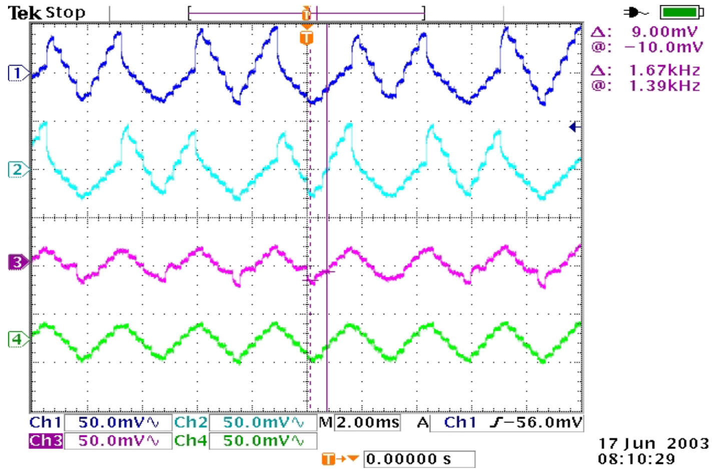


(c)

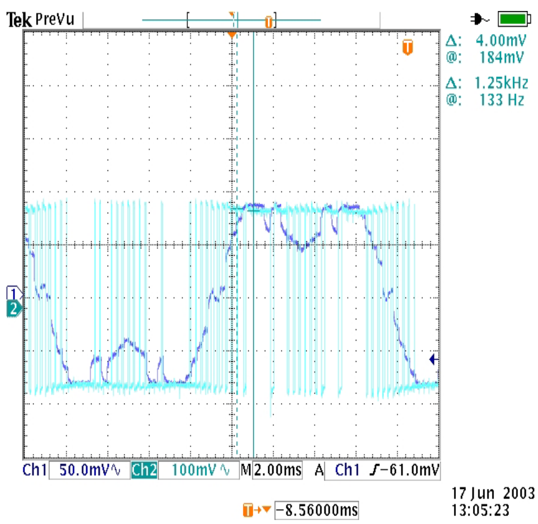


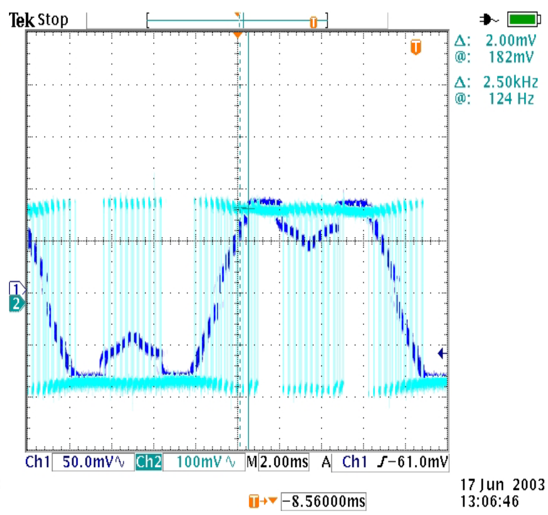
(d)

Figure 5.8 Experimental results: Generation of balanced three-phase voltages using discontinuous modulation scheme for given reference voltages V_{an} = 30 cos (377t), V_{bn} =30 cos (377t - 2π/3), Vcn =30 cos (377t + 2π/3) V_d = 60V. (a) Sationary reference frame q-d voltages, θ, and atan(V_d/V_q) (b) Selection of Sectors for atan(V_d/V_q), (c) Absolute values of t_{dI} t_{dII} t_{dp} t_{dn} (d) Corresponding values of t_{cI}, t_{cII}, t_{cp}, and t_{cn}



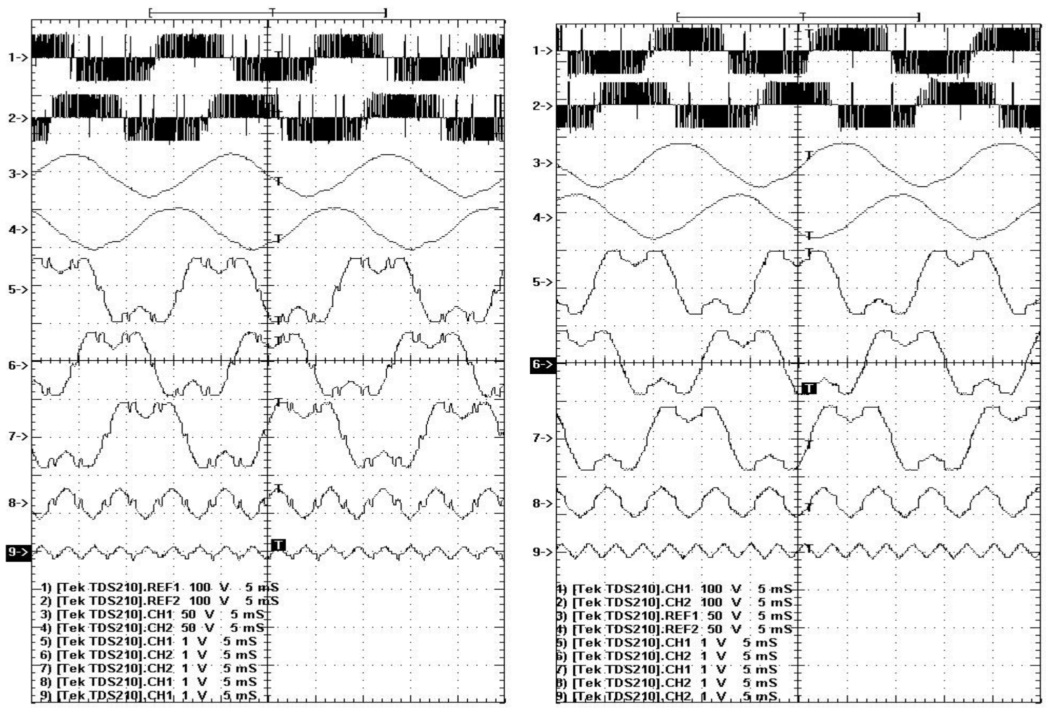
(a)





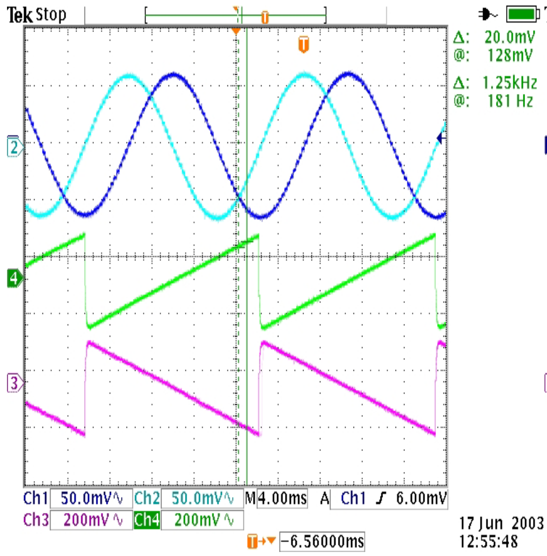
(b) **(c)**

Figure 5.9 Experimental results: Generation of balanced three-phase voltages using discontinuous modulation scheme for given reference voltages V_{an} = 30 cos (377t), V_{bn} =30 cos (377t - 2π/3), Vcn =30 cos (377t + 2π/3) V_d = 60V. (a) Conditions of t_{dI} and t_{dII}, selection of maximum t_{dmax} and minimum t_{dmin} of these values of td (b) Modulating signal and corresponding switching for one of the top switches when (b) $t_d = t_{dmax}$ and (c) td = t_{dmin}

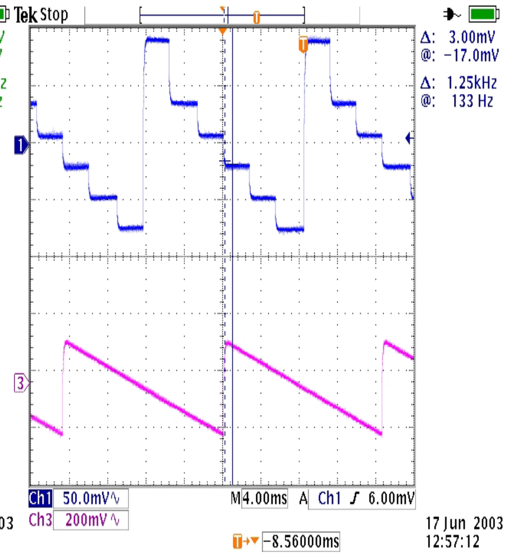


(a) $t_d = t_{dmax}$ **(b) $t_d = t_{dmin}$**

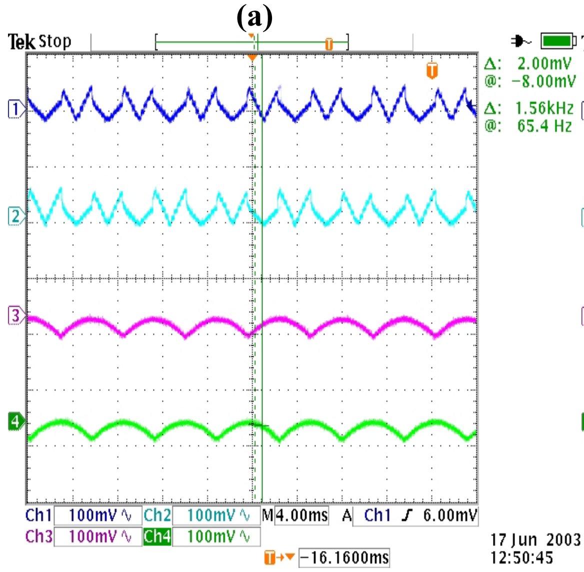
Figure 5.10 Experimental results: Generation of balanced three-phase voltages using discontinuous modulation scheme. V_{an} = 30 cos (377t), V_{bn} =30 cos (377t - 2π/3), Vcn =30 cos (377t + 2π/3) V_d = 60V. (1-2) Line voltages V_{ad}, V_{cd} , (3-4) Filtered line voltages V_{ad}, V_{cd} , (5-8) Modulating signals of the top four devices, S_{ap} , S_{bp} , S_{cp}, S_{dp} , (9) t_d



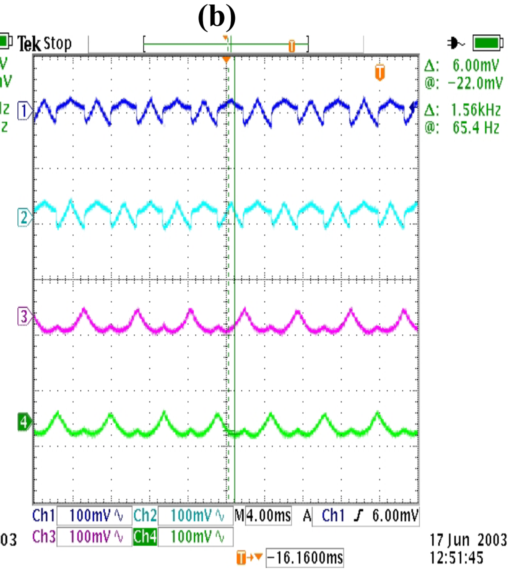
(a)



(b)

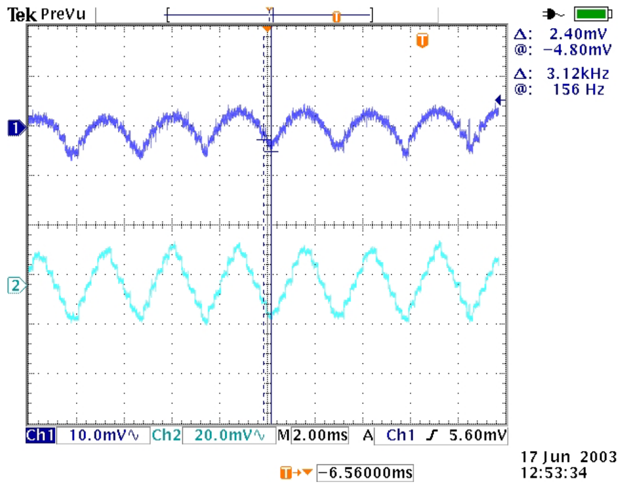


(c)

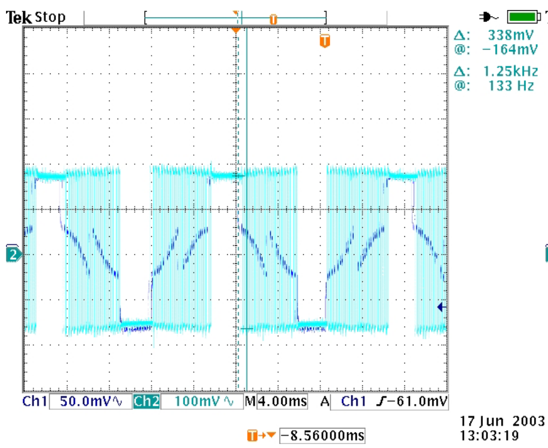


(d)

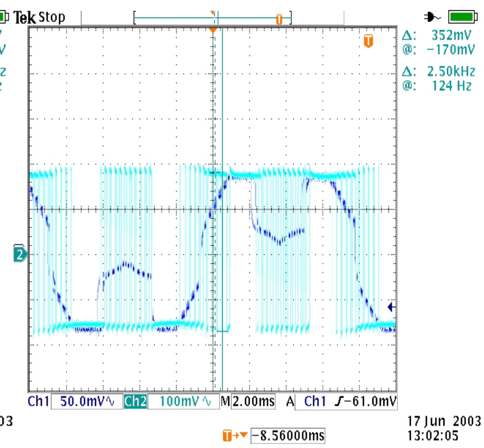
Figure 5.11 Experimental results: Generation of balanced three-phase voltages using discontinuous modulation scheme for given reference voltages V_{an} = 22.5 cos (377t), V_{bn} =22.5cos (377t - 2π/3), V_{cn} = 22.5 cos (377t + 2π/3) V_d = 60V. (a) Sationary reference frame q-d voltages, θ , and atan(V_d/V_q) (b) Selection of Sectors for atan(V_d/V_q), (c) Absolute values of t_{dI} t_{dII} t_{dp} t_{dn} (d) Corresponding values of t_{cI}, t_{cII}, t_{cp}, and t_{cn}



(a)



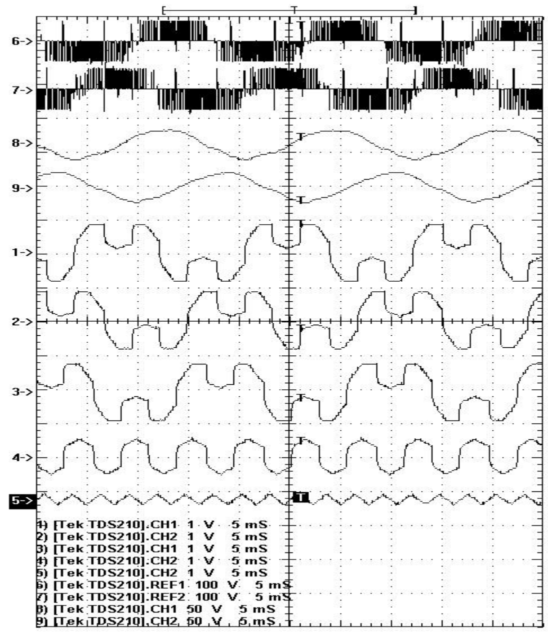
(b)



(c)

Figure 5.12 Experimental results: Generation of balanced three-phase voltages using discontinuous modulation scheme for given reference voltages V_{an} = 22.5 cos (377t), V_{bn} =22.5cos (377t - 2π/3), Vcn = 22.5 cos (377t + 2π/3) V_d = 60V. (a) Conditions of t_{dI} and t_{dII}, selection of maximum t_{dmax} and minimum t_{dmin} of these values of td (b) Modulating signal and corresponding switching for one of the top switches when (b) td = t_{dmax} and (c) td = t_{dmin}

(a) $t_d = t_{dmax}$



(b) $t_d = t_{dmin}$

Figure 5.13 Experimental results: Generation of balanced three-phase voltages using discontinuous modulation scheme. V_{an} = 22.5 cos (377t), V_{bn} =22.5cos (377t - 2π/3), V_{cn} = 22.5 cos (377t + 2π/3) V_d = 60V. (6-7) Line voltages V_{ad}, V_{cd}, (3-4) Filtered line voltages V_{ad}, V_{cd} , (1-4) Modulating signals of the top four devices, S_{ap}, S_{bp} , S_{cp} , S_{dp} , (5) t_d

5.5.2 Synthesis of unbalanced reference voltages for a mild unbalance.

In Figures 5.13 through 5.14 a mild unbalanced generated on phase ‘c’ voltages are synthesized. Figures show types of modulation signals needed for the synthesis of unbalanced voltages and the unevenness in the clamping of the devices to the dc rails. It would appear from these figures that at lower (higher) modulation index, voltage synthesis based on the selection of switching mode combinations that give maximum (minimum) t_d yield the smoother modulation signals and hence less distortion in the synthesized voltage waveforms.

5.5.3 Synthesis of unbalanced reference voltages for severe unbalance.

In Figures 5.15 through 5.16 a severe unbalance on phase ‘a’ and a phase shift in phase ‘c’ voltages are synthesized. Figures show types of modulation signals needed for the synthesis of unbalanced voltages and the unevenness in the clamping of the devices to the dc rails.

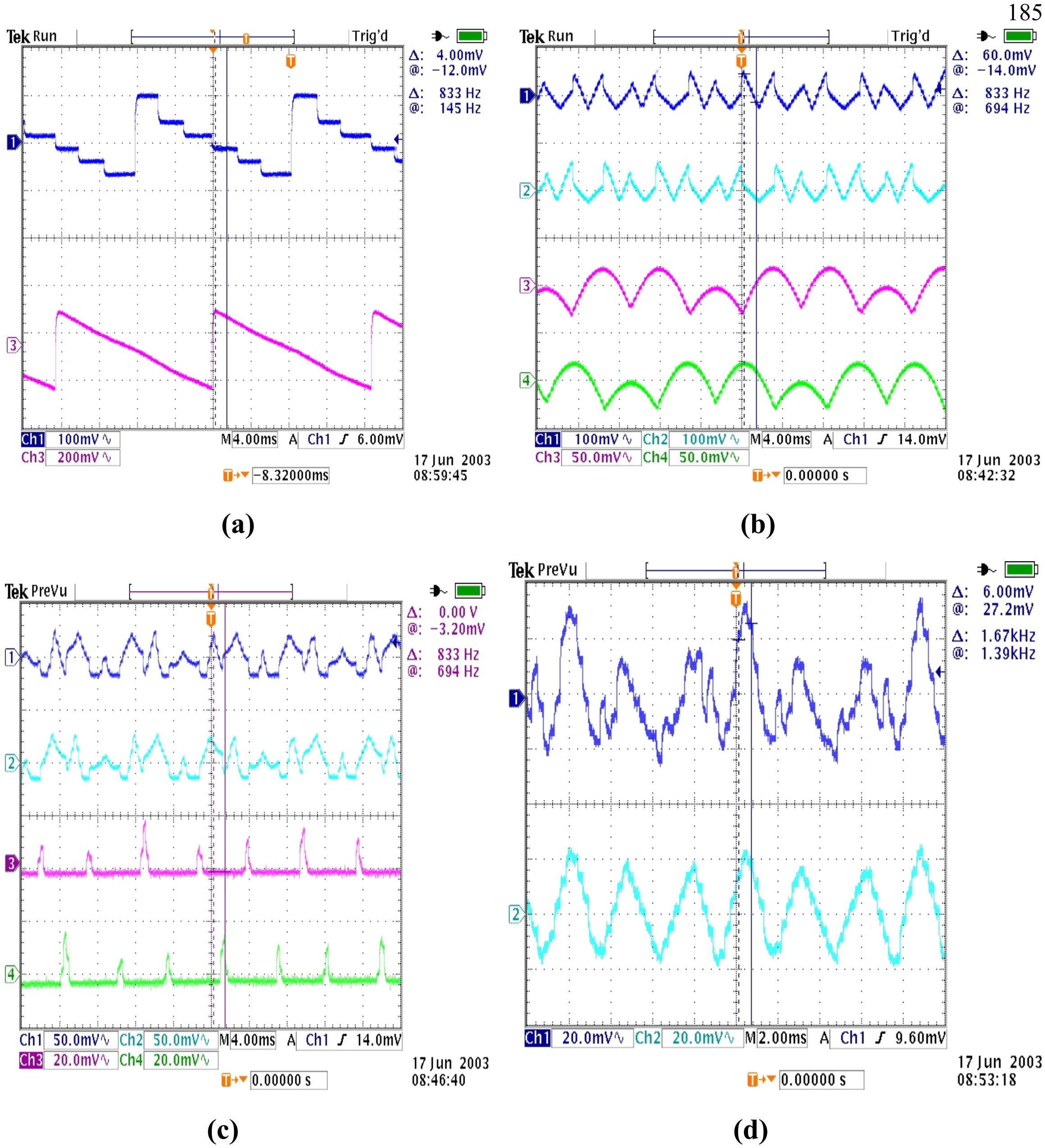
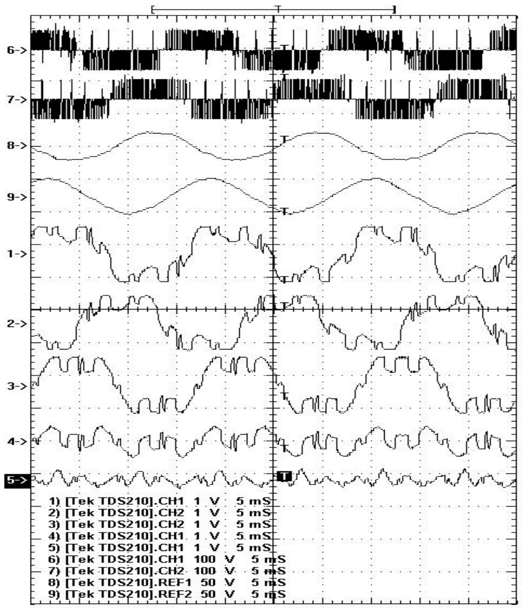
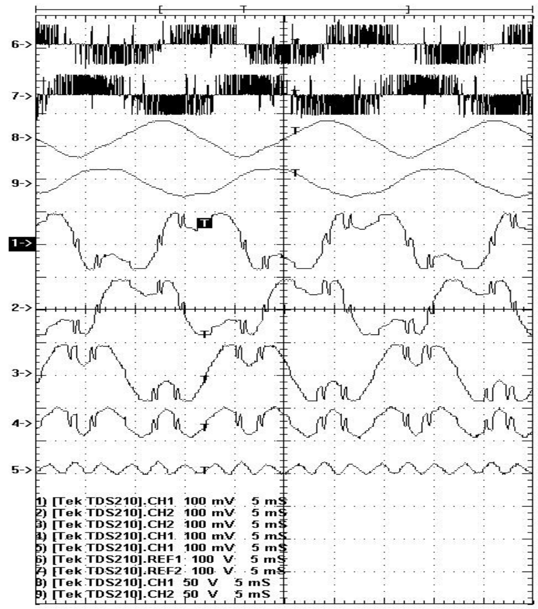


Figure 5.14 Generation of unbalanced three-phase voltages using discontinuous modulation scheme. V_{an} = 30 cos (377t), V_{bn} =30cos (377t - 2π/3), Vcn = 22.5 cos (377t + 2π/3) V_d = 60V. (a) Selection of Sectors for atan(V_d/V_q), (b) Absolute values of t_{dI} t_{dII} t_{dp} t_{dn} (c) Corresponding values of t_{cI}, t_{cII}, t_{cp}, and t_{cn} (d) maximum and minimum of the conditions of t_d

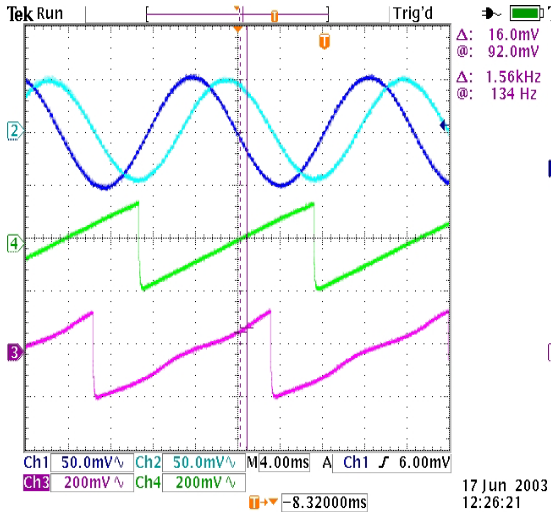


(a) $t_d = t_{dmax}$

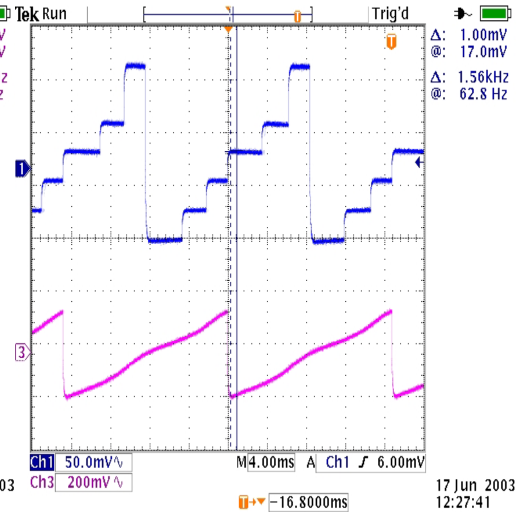


(b) $t_d = t_{dmin}$

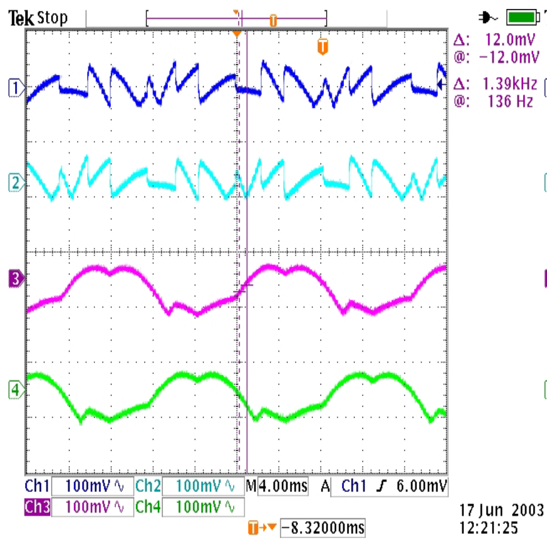
Figure 5.15 Experimental results: Generation of unbalanced three-phase voltages using discontinuous modulation scheme. V_{an} = 30 cos (377t), V_{bn} =30cos (377t - 2π/3), Vcn = 22.5 cos (377t + 2π/3) V_d = 60V. (6-7) Line voltages V_{ad}, V_{cd}, (3-4) Filtered line voltages V_{ad}, V_{cd} , (1-4) Modulating signals of the top four devices, S_{ap} , S_{bp} , S_{cp}, S_{dp} , (5) t_d



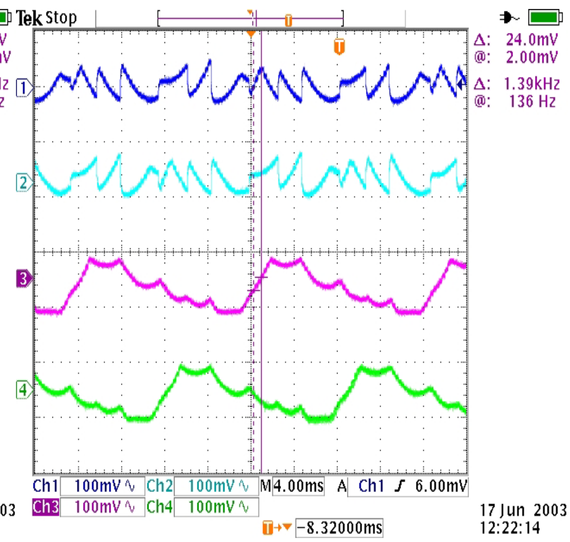
(a)



(b)

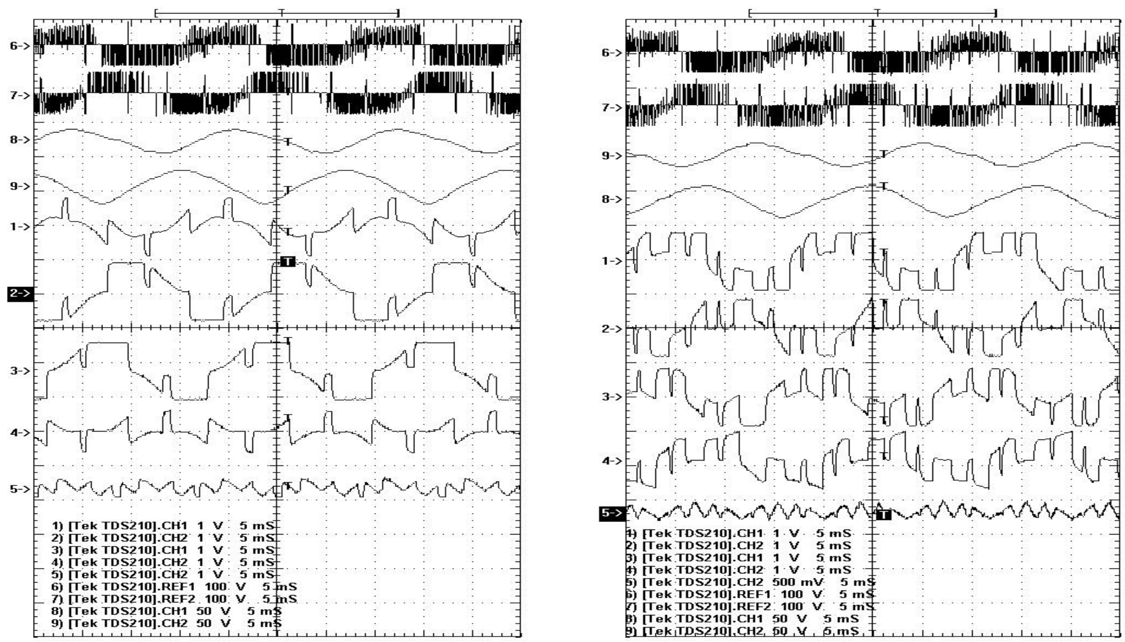


(c)



(d)

Figure 5.16 Generation of unbalanced three-phase voltages using discontinuous modulation scheme. V_{an} = 10 cos (377t), V_{bn} =30cos (377t - 2π/3), V_{cn} = 30 cos (377t + π) V_d = 60V. (a) Selection of Sectors for atan(V_d/V_q), (b) Absolute values of t_{dI} t_{dII} t_{dp} t_{dn} (c) Corresponding values of t_{cI}, t_{cII}, t_{cp}, and t_{cn} (d) maximum and minimum of the conditions of t_d



(a) $t_d = t_{dmax}$ **(b) $t_d = t_{dmin}$**

Figure 5.17 Experimental results: Generation of unbalanced three-phase voltages using discontinuous modulation scheme. V_{an} = 10 cos (377t), V_{bn} =30cos (377t - 2π/3), Vcn = 30 cos (377t + π) V_d = 60V. (6-7) Line voltages V_{ad}, V_{cd}, (3-4) Filtered line voltages V_{ad}, V_{cd}, (1-4) Modulating signals of the top four devices, S_{ap}, S_{bp}, S_{cp}, S_{dp}, (5) t_d